Szymon Głąb

Topological size of sets in function spaces defined by pointwise product and convolution

with Filip Strobin

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Balcerzak & Wachowicz – first results

There are topological function spaces X such that a natural multiplication $f \cdot g$ is defined, but its result is not necessarily element of X. A general problem is whether the set of those pairs (f,g) for which $f \cdot g \notin X$ is topologically large.

Balcerzak & Wachowicz, 2000

The following sets

(i) $\{(x,y) \in c_0 \times c_0 : (\sum_{i=1}^{n} x_i y_i)_{n=1}^{\infty} \text{ is bounded}\}$,

(ii) $\{(f,g) \in L^1[0,1] \times L^1[0,1] : \int_0^1 |f \cdot g| < \infty\}$

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Jachymski's extension of the classical Banach–Steinhaus theorem

A function $\varphi : X \to \mathbb{R}_+$ is called L-subadditive, $L \geq 1$, if $\varphi(x + y) \leq L(\varphi(x) + \varphi(y))$ for any $x, y \in X$.

Jachymski, 2005, an extension of the classical Banach–Steinhaus theorem

Given $k \in \mathbb{N}$, let $X_1, \ldots, X_k$ be Banach spaces, $X = X_1$ if $k = 1$, and $X = X_1 \times \ldots \times X_k$ if $k > 1$. Assume that $L \geq 1$, $F_n : X \to \mathbb{R}_+$ $(n \in \mathbb{N})$ are lower semicontinuous and such that all functions $x_i \mapsto F_n(x_1, \ldots, x_k)$ $(i = 1, \ldots, k)$ are L-subadditive and even. Let $E = \{x \in X : (F_n(x))_{n=1}^\infty$ is bounded$\}$. Then the following statements are equivalent:

(i) E is meager;

(ii) $E \neq X$;

(iii) $\sup\{F_n(x) : n \in \mathbb{N},\ ||x|| \leq 1\} = \infty$.

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Further improvement of Jachymski's result

Let X be a metric space. $B(x,R)$ stands for the ball with a radius R centered at a point x. Let $c \in (0,1]$. We say that $M \subset X$ is *c-lower porous*, if

$$\forall x \in M \quad \liminf_{R \to 0^+} \frac{\gamma(x,M,R)}{R} \geq \frac{c}{2},$$

where

$$\gamma(x,M,R) = \sup\{r \geq 0 : \exists z \in X \quad B(z,r) \subset B(x,R) \setminus M\}.$$

At first, we were interested in a further generalization of Jachymski's theorem changing meagerness by σ-porosity. It is not possible. To see it, consider the following set:

$$E = \left\{ x \in \mathbb{R} : \left(\sum_{k=1}^{n} \frac{|\sin(k!\pi x)|}{k} \right)_{n=1}^{\infty} \text{ is bounded} \right\}.$$

Using Jachymski's Theorem for $F_n(x) = \sum_{k=1}^{n} |\sin(k!\pi x)|/k$ we obtain that this set is meager ($E \neq \mathbb{R}$ since it is of measure zero) and E is not σ-upper porous (it is well known example of such set).

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L^p spaces, G & Strobin, 2010, J. Math. Anal. Appl.

Let (X, Σ, μ) be a measure space and $p \in (0, \infty]$. For $n \in \mathbb{N}$ and $p_1, \dots, p_n, r \in (0, \infty]$ we define the set

$$E_r^{(p_1,\dots,p_n)} = \{(f_1, \dots, f_n) \in L^{p_1} \times \dots \times L^{p_n} : f_1 \cdot \dots \cdot f_n \in L^r\}.$$

Hölder inequality

If $\frac{1}{r} = \frac{1}{p_1} + \dots + \frac{1}{p_n}$, then $E_r^{(p_1,\dots,p_n)} = L^{p_1} \times \dots \times L^{p_n}$.

$\Sigma_+ := \{A \in \Sigma : 0 < \mu(A) < \infty\}$,

Case when $E_r^{(p_1,\dots,p_n)}$ is large

If one of the following conditions holds:

(i) $\sup\{\mu(A) : A \in \Sigma_+\} < \infty$ and $0 < \frac{1}{p_1} + \dots + \frac{1}{p_n} < \frac{1}{r}$;

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Assume that one of the following conditions holds:

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Then for any $u > 0$, the set

$$E_u = \{(f_1, \dots, f_n) \in L^{p_1} \times \dots \times L^{p_n} : ||f_1 \cdot \dots \cdot f_n||_r \leq u\}$$

is c-lower porous, where $c = c(p_1, \dots, p_n)$. In particular, the set $E_r^{(p_1,\dots,p_n)}$ is σ-c-lower porous.

Dichotomy

Either $E_r^{(p_1,\dots,p_n)}$ is σ-c-lower porous **or** $E_r^{(p_1,\dots,p_n)} = L^{p_1} \times \dots \times L^{p_n}$.

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Lorentz spaces – G., Strobin & Chan Woo Yang, Cent. Eur. J. Math. 2013, 11(7), 1228–1242

Let (X, Σ, μ) be a measure space. Let $p, q \in (0, \infty]$ be such that if $p = \infty$, then also $q = \infty$. A Lorentz space $\mathbf{L}^{p,q}(X, \Sigma, \mu)$ ($\mathbf{L}^{p,q}$ in short) is the space of all measurable functions with a finite quasinorm (the triangle inequality does not hold)

$$\|f\|_{p,q} := \begin{cases} \left(\int_0^\infty p\mu(\{x : |f(x)| > \lambda\})^{\frac{q}{p}} \lambda^{q-1} d\lambda\right)^{\frac{1}{q}}, & \text{if } q < \infty; \\ \sup_{\lambda > 0} \lambda\mu(\{x : |f(x)| > \lambda\})^{\frac{1}{p}}, & \text{if } p < \infty \text{ and } q = \infty; \\ \operatorname{supess} |f|, & \text{if } p = q = \infty. \end{cases}$$

$\|f\|_{p,p} = \|f\|_p = (\int_X |f|^p)^{1/p}$ for $p \geq 1$.

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$$E_{p,q}^{(p_1,q_1,\dots,p_n,q_n)} := \{(f_1, \dots, f_n) \in \mathbf{L}^{p_1,q_1} \times \dots \times \mathbf{L}^{p_n,q_n} : f_1 \cdots f_n \in \mathbf{L}^{p,q}\}.$$

Theorem

Let $n \in \mathbb{N}$ and $\mathbf{L}^{p,q}, \mathbf{L}^{p_1,q_1}, \dots, \mathbf{L}^{p_n,q_n}$ be Lorentz spaces such that if $p < \infty$, then $\frac{1}{p} \neq \frac{1}{p_1} + \dots + \frac{1}{p_n}$. Then the following conditions are equivalent:

(a) the set $E_{p,q}^{(p_1,q_1,\dots,p_n,q_n)}$ is σ-α-lower porous in $\mathbf{L}^{p_1,q_1} \times \dots \times \mathbf{L}^{p_n,q_n}$ for some $\alpha > 0$;

(b) $E_{p,q}^{(p_1,q_1,\dots,p_n,q_n)} \neq \mathbf{L}^{p_1,q_1} \times \dots \times \mathbf{L}^{p_n,q_n}$;

(c) one of the conditions holds:

(i) $\Sigma_+ \neq \emptyset$ and $\inf\{\mu(A) : A \in \Sigma_+\} = 0$ and $\frac{1}{p_1} + \dots + \frac{1}{p_n} > \frac{1}{p}$;

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(iii) $\mu(X) = \infty$ and $\min\{p_1, \dots, p_n\} = \infty$ and $p < \infty$.

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Lorentz spaces – G., Strobin & Chan Woo Yang, Cent. Eur. J. Math. 2013, 11(7), 1228–1242

$$E_{p,q}^{(p_1,q_1,\dots,p_n,q_n)} := \{(f_1,\dots,f_n) \in \mathbf{L}^{p_1,q_1} \times \dots \times \mathbf{L}^{p_n,q_n} : f_1 \cdots f_n \in \mathbf{L}^{p,q}\}.$$

Theorem

Let $n \in \mathbb{N}$ and $\mathbf{L}^{p,q}, \mathbf{L}^{p_1,q_1}, \dots, \mathbf{L}^{p_n,q_n}$ be Lorentz spaces such that if $p < \infty$, then $\frac{1}{p} \neq \frac{1}{p_1} + \dots + \frac{1}{p_n}$. Then the following conditions are equivalent:

(a) the set $E_{p,q}^{(p_1,q_1,\dots,p_n,q_n)}$ is σ-α-lower porous in $\mathbf{L}^{p_1,q_1} \times \dots \times \mathbf{L}^{p_n,q_n}$ for some $\alpha > 0$;

(b) $E_{p,q}^{(p_1,q_1,\dots,p_n,q_n)} \neq \mathbf{L}^{p_1,q_1} \times \dots \times \mathbf{L}^{p_n,q_n}$;

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Orlicz spaces – Strobin, Math. Slovaca 66 (2016), no. 1, 245–256.

Let $\psi : [0,\infty) \to [0,\infty)$ be a continuous non-decreasing and convex with $\psi(0) = 0$, $\psi(t) > 0$ for $t > 0$, and $\lim_{t\to\infty} \psi(t) = \infty$. It is so-called *Young function*.

$L^\psi(X,\Sigma,\mu)$ (L^ψ in short) is the set of all measurable functions f defined on X such that $\int_X \psi(v|f|)d\mu < \infty$ for some $v > 0$. Then L^ψ is a Banach space with the following norm

$$\|f\|_\psi := \inf\{u : \int_X \psi(\frac{|f|}{u})d\mu \leq 1\}.$$

The space L^ψ is called the Orlicz space.
If $p \geq 1$ and $\psi(t) = t^p$, then $L^\psi = L^p$.

$$E_\psi^{(\psi_1,\dots,\psi_n)} := \{(f_1,\dots,f_n) \in \mathbf{L}^{\psi_1} \times \dots \times \mathbf{L}^{\psi_n} : f_1 \cdots f_n \in \mathbf{L}^\psi\}.$$

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Theorem

Let $\Sigma_+ \neq \emptyset$ and $\mathbf{L}^{\psi}, \mathbf{L}^{\psi_1}, \dots, \mathbf{L}^{\psi_n}$ be Orlicz spaces. Assume that one of the conditions holds:

1. $\lim_{t\to 0} \frac{\psi^{-1}(t)}{\psi_1^{-1}(t)\cdots\psi_n^{-1}(t)}$ and $\lim_{t\to\infty} \frac{\psi^{-1}(t)}{\psi_1^{-1}(t)\cdots\psi_n^{-1}(t)}$ exists (finite or infinite)
2. the set $F = \{\frac{1}{\mu(A)} : A \in \Sigma_+\}$ is an interval.

Then the following conditions are equivalent:

(a) the set $E_{\psi}^{(\psi_1,\dots,\psi_n)}$ is σ-$\frac{2}{n+1}$-lower porous in $\mathbf{L}^{\psi_1} \times \dots \times \mathbf{L}^{\psi_n}$;

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If $\psi(t) = t^p$, $\psi_i(t) = t^{p_i}$, then

$$\lim_{t\to 0} \frac{\psi^{-1}(t)}{\psi_1^{-1}(t)\cdots\psi_n^{-1}(t)} = 0 \iff \lim_{t\to 0} \frac{t^{\frac{1}{p}}}{t^{\frac{1}{p_1}}\cdots t^{\frac{1}{p_n}}} = 0 \iff \lim_{t\to 0} t^{\frac{1}{p}-(\frac{1}{p_1}+\cdots+\frac{1}{p_n})} = 0$$

$$\iff \frac{1}{p} > \frac{1}{p_1} + \cdots + \frac{1}{p_n}$$

and

$$\lim_{t\to\infty} \frac{\psi^{-1}(t)}{\psi_1^{-1}(t)\cdots\psi_n^{-1}(t)} = 0 \iff \frac{1}{p} < \frac{1}{p_1} + \cdots + \frac{1}{p_n}.$$

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G. & Strobin, Porosity and the L^p-conjecture, Arch. Math. 95 (2010), 583–592.

Assume that G is a locally compact group and let μ be a left–invariant Haar measure on G. If f, g are two measurable functions, $x \in G$, then the convolution of f and g in the point x is given by

$$f \star g(x) = \int_G f(y) g(y^{-1}x)\, d\mu(y).$$

Assume that $p > 1$. The famous L^p-conjecture, stated by Zelazko and Rajagopalan in 1960's, asserts that if for all $f, g \in L^p$, $f \star g \in L^p$ (that is, $f \star g$ is defined almost everywhere on G and belongs to L^p), then G is compact. During the next 30 years this conjecture had been established in special cases, and, finally, in 1990 Saeki proved the L^p-conjecture in its general form. Abtahi, Nasr–Isfahani and Rejali in 2007 proved that if G is not compact, then there exist functions $f, g \in L^p$ such that $f \star g$ is not well defined in the sense that there exists a set $K \subset G$ of a positive measure such that for any $x \in K$, $f \star g(x) = \infty$.

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Theorem

Assume that G is locally compact but not compact topological group and μ is a Haar measure on G. If $p, q > 1$ are such that $\frac{1}{p} + \frac{1}{q} < 1$, then

(i) For every compact subset $K \subset G$, the set

$$E_K = \{(f, g) \in L^p \times L^q : \exists x \in K \;\; f \star g(x) \text{ is finite or infinite}\}$$

is σ-c-lower porous for some $c > 0$.

(ii) If G is σ-compact, then the set

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is σ-lower porous.

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G. & Strobin, Porosity and the L^p-conjecture, Arch. Math. 95 (2010), 583–592.

Theorem

Assume that G is locally compact but not compact topological group and μ is a Haar measure on G. If $p, q > 1$ are such that $\frac{1}{p} + \frac{1}{q} < 1$, then

(i) For every compact subset $K \subset G$, the set

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Recent results

I. Akbarbaglu, G. , S. Maghsoudi, F. Strobin, Topological size of some subsets in certain Calderón-Lozanowskiĭ spaces, Adv. Math. 312 (2017), 737–763.

Calderón-Lozanowskiĭ spaces, Adv. Math. 312 (2017), 737–763.

For any Young function φ we define:

$$a_\varphi = \sup\{x \in \mathbb{R} : \varphi(x) = 0\} \quad \text{and} \quad b_\varphi = \sup\{x \in \mathbb{R} : \varphi(x) < \infty\}.$$

Let (Ω, Σ, μ) be a complete σ-finite measure space. A Banach space $E = (E, \|\cdot\|_E)$ is called *a Banach ideal space* on Ω if E is a linear subspace of $L^0(\Omega)$ with the ideal property:
if $f \in E, g \in L^0(\Omega)$ and $|g(t)| \leq |f(t)|$ for μ-almost all $t \in \Omega$, then $g \in E$ and $\|g\|_E \leq \|f\|_E$.
Let $I_\varphi : L^0(\Omega) \to [0, \infty]$ be a semimodular defined by

$$I_\varphi(f) = \begin{cases} \|\varphi(|f|)\|_E & \text{if} \quad \varphi(|f|) \in E, \\ \infty & \text{otherwise.} \end{cases}$$

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$$E_\varphi = \{f \in L^0(\Omega) : I_\varphi(cf) < \infty \text{ for some } c > 0\}$$

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The notion of Calderón-Lozanowskiĭ space E_φ generalize some known notions:

1. If $E = L^1(\Omega)$, then E_φ is the Orlicz space $L^\varphi(\Omega)$ equipped with the Luxemburg norm.
2. If E is a Lorentz function (sequence) space, then E_φ is the corresponding Orlicz-Lorentz function (sequence) space equipped with the Luxemburg norm.
3. If $\varphi(t) = t^p$, $1 \leq p < \infty$, then E_φ is in this case the p convexification $E^{(p)}$ of E with the norm $\|f\|_{E^{(p)}} = \||f|^p\|_E^{1/p}$.
4. If $\varphi(t) = 0$ for $t \in [0,1]$ and $\varphi(t) = \infty$ otherwise, then $E_\varphi = L^\infty(\Omega)$ and the corresponding norms are equal.

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Calderón-Lozanowskiĭ spaces – pointwise multiplication

Let (Ω, Σ, μ) be a complete σ-finite measure space and E be a Banach ideal in $L^0(\Omega)$ such that $\chi_A \in E$ for every $A \in \Sigma_+$.

Theorem

Let E_{φ_1}, E_{φ_2}, E_{φ_3} be Calderón-Lozanowskiĭ spaces with $\Sigma_+ \neq \emptyset$. Assume that $a_{\varphi_3} = 0$ and for any $\varepsilon > 0$ there is $A \in \Sigma_+$ such that $\frac{1}{\|\chi_A\|_E} \leq \min\{\varphi_1(b_{\varphi_1}), \varphi_2(b_{\varphi_2})\}$ and

$$\frac{\|\chi_A\|_{E_{\varphi_1}} \cdot \|\chi_A\|_{E_{\varphi_2}}}{\|\chi_A\|_{E_{\varphi_3}}} = \frac{\varphi_3^{-1}\left(\frac{1}{\|\chi_A\|}\right)}{\varphi_1^{-1}\left(\frac{1}{\|\chi_A\|}\right) \cdot \varphi_2^{-1}\left(\frac{1}{\|\chi_A\|}\right)} \leq \varepsilon. \quad (1)$$

Then the set $F = \{(f_1, f_2) \in E_{\varphi_1} \times E_{\varphi_2} : f_1 \cdot f_2 \in E_{\varphi_3}\}$ is $\sigma - \frac{2}{3}$-lower porous.

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A Banach ideal space E is called *order continuous* if for every $f \in E$ and every sequence $\{A_n\}$ satisfying $A_n \downarrow \emptyset$ (that is $A_n \supset A_{n+1}$ and $\mu\left(\bigcap_{n=1}^{\infty} A_n\right) = 0$), we have that $\|f\chi_{A_n}\|_E \downarrow 0$.

Theorem

Let φ_1, φ_2 and φ_3 be Young functions with $b_{\varphi_3} = \infty$ and E be a Banach ideal space with order continuous norm. If there exists $(h, k) \in E_{\varphi_1} \times E_{\varphi_2}$ such that $h \cdot k \notin E_{\varphi_3}$, then the set

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Calderón-Lozanowskiĭ spaces – convolution

let G be a locally compact group with a fixed left Haar measure λ. Let E be a Banach ideal in $L^0(G)$ such that

(a) if $f_n \nearrow f$ for some nonnegative functions $f_n \in E$, $n \in \mathbb{N}$ and $f \in L^0(G)$, then $\|f_n\|_E \to \|f\|_E$ provided $f \in E$, and $\|f_n\|_E \to \infty$ if $f \notin E$.

(b) if $V \subset G$ and $\lambda(V) < \infty$, then $\chi_V \in E$;

(c) if $V \subset G$ and $\lambda(V) < \infty$, then there is $C_V < \infty$ such that $\int_V |f| d\lambda \leq C_V \|f\|_E$ for every $f \in E$.

$f, g \in L^0(G)$ are called *equimeasurable*, if $\lambda(\{x \in G : |f(x)| > t\}) = \lambda(\{x \in G : |g(x)| > t\})$ for every $t \geq 0$.
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(b) if $V \subset G$ and $\lambda(V) < \infty$, then $\chi_V \in E$;

(c) if $V \subset G$ and $\lambda(V) < \infty$, then there is $C_V < \infty$ such that $\int_V |f| d\lambda \leq C_V \|f\|_E$ for every $f \in E$.

$f, g \in L^0(G)$ are called *equimeasurable*, if $\lambda(\{x \in G : |f(x)| > t\}) = \lambda(\{x \in G : |g(x)| > t\})$ for every $t \geq 0$.
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Note that if $\lambda(V) = \lambda(U)$, then χ_V and χ_U are equimeasurable, hence $\|\chi_V\|_E = \|\chi_U\|_E$. Thus there exists a function $\xi_E : [0, \infty) \to [0, \infty)$ such that for every measurable $V \subset G$ with $\lambda(V) < \infty$

$$\xi_E(\lambda(V)) := \|\chi_V\|_E.$$

The function ξ_E (which is uniquely determined just on the range of λ), is called the *fundamental function of E*.

Finally, we make the following assumptions on the fundamental function:

(e) The fundamental function ξ_E is continuous at 0, that is, $\lim_{t\to 0} \xi_E(t) = 0$.

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and let φ_i, $i = 1, 2, 3$ be Young functions with $\varphi_i(b_{\varphi_i}) > 0$, for $i = 1, 2, 3$ and

$$\liminf_{x\to 0} \frac{\varphi_1^{-1}(x)\varphi_2^{-1}(x)}{x\varphi_3^{-1}(x)} = \infty.$$

If G is non-compact, then the set $F := \{(f, g) \in E_{\varphi_1} \times E_{\varphi_2} : |f| * |g| \in E_{\varphi_3}\}$ is of first category in $E_{\varphi_1} \times E_{\varphi_2}$.

Locally compact group G having polynomial growth satisfies the condition ($\star$).
It is an extension of a result from [A. Kamińska and J. Musielak, On convolution operator in Orlicz spaces, *Rev. Mat. Complut.* **2** (1989), 157–178.] where it is shown that $F \neq E_{\varphi_1} \times E_{\varphi_2}$ in the case of Orlicz spaces E_{φ_i} for Abelian groups G.

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Let G be a locally compact group that satisfies the condition

($\star\star$) *for every compact neighbourhood* V *of the identity element of* G *there exist* $\kappa > 1$ *and a sequence* $(U_n)_{n\in\mathbb{N}}$ *contained in* V *with* $\lim_{n\to\infty} \lambda(U_n) = 0$ *and* $\lambda(U_n^{-1}U_n) \leq \kappa\lambda(U_n)$*;*

and let φ_i, $i = 1, 2, 3$ be Young functions such that

$$\liminf_{x\to\infty} \frac{\varphi_1^{-1}(x)\varphi_2^{-1}(x)}{x\varphi_3^{-1}(x)} = \infty.$$

Then the set

$$F = \{(f, g) \in L^{\varphi_1}(G) \times L^{\varphi_2}(G) : |f| * |g| \in L^{\varphi_3}(G)\}.$$

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Calderón-Lozanowskiĭ spaces – convolution

Locally compact group G is *amenable* whenever it fulfills so-called Leptin condition, that is for every compact subset U of G and any $\epsilon > 0$ there exists a compact subset V in G of positive measure such that $\lambda(UV) < (1+\epsilon)\lambda(V)$.

Theorem

Let G be an amenable locally compact group, φ a Young function with $\lim_{t\to 0} \varphi(t)/t = 0$, $\varphi(b_\varphi) > 0$ and ψ be a Young function with $\psi(b_\psi) = \infty$. If G is non-compact, then the set

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The above generalize the main result of [H. Hudzik, A. Kamińska and J. Musielak, On some Banach algebras given by a modular (1985)] from abelian locally compact groups to amenable ones in the context of Orlicz spaces. The amenability hypothesis cannot be dropped – R.A. Kunze and E.M. Stein (1960) show that the multiplication group of real matrices with determinant 1, $G = SL(2,\mathbb{R})$, satisfies $L^p(G) * L^2(G) \subset L^2(G)$ for $1 \le p < 2$.

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$$\liminf_{x \to 0} \frac{\varphi^{-1}(x)\psi^{-1}(x)}{x} = \infty.$$

(1.) If E is a real space, then for every compact set V with $\lambda(V) > 0$, the set

$$F_V = \{(f, g) \in E_\varphi \times E_\psi : f * g(x) \text{ is well defined in some point } x \in V\}$$

is of first category in $E_\varphi \times E_\psi$.
(2.) If E is complex, then for every compact set V with $\lambda(V) > 0$, the set

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Calderón-Lozanowskiĭ spaces – convolution

For each $x \in G$, $\lambda_x(A) = \lambda(Ax)$ is a left invariant regular Borel measure on G. The uniqueness of the left Haar measure implies that for each $x \in G$ there is a positive number, say $\Delta(x)$, such that $\lambda_x = \Delta(x)\lambda$. The function $\Delta : G \to (0, \infty)$ is called the modular function of G. Δ is a continuous homomorphism on G. The group G is called *unimodular* whenever $\Delta = 1$. In this case, the left Haar measure and the right Haar measure coincide.

Theorem

Assume that G is a non-unimodular locally compact group and φ, ψ are Young functions with $\lim_{t\to 0} \varphi(t)/t = 0$, $\varphi(b_\varphi) > 0$ and $\psi(b_\psi) > 0$. For every compact set V with $\lambda(V) > 0$, the set

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Side results

S & Strobin, Dichotomies for $C_0(X)$ and $C_b(X)$ spaces, Czechoslovak Math. J. 63(1), (2013), 91–105.

Theorem

Assume that (X, μ) is a topological measure space which is inner regular and such that the topological space X is locally compact and σ-compact. Let $h \in \mathbf{L}^1_{loc}$ and let (D_n) be a sequence of measurable subsets of X such that $\sup_{n\in\mathbb{N}} \int_{D_n} |h| d\mu = \infty$. Then the set

$$E^0_{h,(D_n)} := \left\{ (f, g) \in \mathbf{C}_0 \times \mathbf{C}_0 : \left(\int_{D_n} fgh \, d\mu \right)_{n=1}^{\infty} \text{ is bounded} \right\}$$

is σ-strongly ball porous.

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The set $E = \{(x, y) \in c_0 \times c_0 : (\sum_{i=1}^{n} x_i y_i)_{n=1}^{\infty} \text{ is bounded}\}$ is meager.

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Side results

G& Strobin, Spaceability of sets in $L_p \times L_q$ and $C_0 \times C_0$, J. Math. Anal. Appl. 440 (2016), no. 2, 451–465.

Theorem

Assume that one of the following conditions hold:

(i) $0 < \frac{1}{p} + \frac{1}{q} < \frac{1}{r}$ and $\sup\{\mu(A) : A \in \Sigma, \mu(A) < \infty\} = \infty$;

(ii) $\frac{1}{p} + \frac{1}{q} > \frac{1}{r}$ and $\inf\{\mu(A) : A \in \Sigma, \mu(A) > 0\} = 0$.

Then the set $E = \{(f,g) \in L^p \times L^q : fg \notin L^r\}$ is speceable in $L^p \times L^q$.

Theorem

Let G be a locally compact non-compact topological group. Let K be a fixed compact symmetric neighborhood of the identity element of G. Let $\infty > p, g > 1$ be such that $\frac{1}{p} + \frac{1}{q} < 1$. Then the set

$$E = \{(f,g) \in L^p \times L^q : \forall_{x \in K} \ (f \star g(x) = \infty \text{ or } f \star g(x) \text{ does not exists})\}$$

is spaceable.

Further questions

1. Is a quantitative version of Saeki Theorem (the solution for L^p-conjecture) true?
2. Are the spaceability results, such as in the previous slide, true for Calderón-Lozanowskiĭ spaces?