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TRANSITIVE LIE ALGEBROIDS OF RANK 1 AND LOCALLY CONFORMAL SYMPLECTIC STRUCTURES

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Abstract. The aim of the paper is the study of transitive Lie algebroids with the trivial 1-rank adjoint bundle of isotropy Lie algebras $\boldsymbol{g} \cong M \times \mathbb{R}$. We show that a locally conformal symplectic (l.c.s.) structure defines such a Lie algebroid, so our algebroids are a natural generalisation of l.c.s. structures. We prove that such a Lie algebroid has the Poincaré duality property for the Lie algebroid cohomology (TUIO-property) if and only if the top dimensional cohomology space is non trivial. Moreover, if an algebroid is defined by a l.c.s. structure, then this algebroid is a TUIO-Lie algebroid if and only if the associated l.c.s. structure is a globally conformal symplectic structure.

1. Introduction

Lie algebroids play important role in geometry. They appear as naturally associated to many well-known and important geometrical objects like G-principal fibre bundles, TC-foliations, Poisson and Jacobi manifolds or singular foliations, cf. [P], [M], [MO], [K1], [K2], [K3], [C-D-W], [K-S]. Moreover, a Lie algebroid is naturally associated to any Lie pseudogroup. The cohomology ring defined by a Lie algebroid is a suitable setting characteristic classes of such a geometrical structure - they generalise the classical theory of characteristic classes, cf. [K2], [K4], [K7], [C], [F]. Therefore, we think, that it is important to study the properties of this cohomology ring, in particular the Poincaré duality property. In this respect the second author proved the following theorem, [K6].

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Theorem 1.1. *Let $A = (A, [\![\cdot,\cdot]\!], \gamma)$ be a transitive Lie algebroid on a connected oriented m-manifold M and let*

$$0 \longrightarrow \boldsymbol{g} \longrightarrow A \xrightarrow{\gamma} TM \longrightarrow 0. \tag{1.1}$$

be the Atiyah sequence of A. Assume that

(a) *the isotropy Lie algebras $\boldsymbol{g}_{|x}$ are unimodular,*
(b) *there exists a non-singular cross-section $\varepsilon \in \operatorname{Sec} \bigwedge^n \boldsymbol{g}$, $n = \operatorname{rank} \boldsymbol{g}$, invariant with respect to the adjoint representation;*

then the family of bilinear homomorphisms

$$P^k : H^k_A(M) \times H^{n+m-k}_{A,c}(M) \longrightarrow \mathbb{R}, \quad ([\Phi], [\Psi]) \longmapsto \int_M \fint_{(A,\varepsilon)} \Phi \wedge \Psi,$$

($k = 0, 1, ..., n+m$) yields a Poincaré duality $P : H_A(M) \times H_{A,c}(M) \to \mathbb{R}$ (for the fibre integral $\fint_{(A,\varepsilon)}$, see [K5]*). In particular, if M is, in addition, compact, then the cohomology algebra $H^*_A(M)$ of real A-differential forms is finite-dimensional and satisfies the Poincaré duality.*

The transitive Lie algebroid fulfilling properties (a) and (b) from the above theorem is called a *TUIO-Lie algebroid* [K5] (*transitive unimodular invariantly oriented*). For examples of TUIO-Lie algebroids of some principal bundles, nonclosed Lie subgroups and TP-foliations see [K6].

The second author discovered a very important fact that for the Lie algebroid $A = A(M, \mathcal{F})$ associated to a TC-foliation $\mathcal{F}$ on a manifold M there is an isomorphism of differential graded algebras $\Omega(A(M, \mathcal{F})) \cong \Omega_b(M, \mathcal{F})$ [K6, Th.6.10]. Therefore the cohomology algebra $H_{A(M,\mathcal{F})}(W)$ of the Lie algebroid $A(M, \mathcal{F})$ satisfies the Poincaré duality if and only if the basic cohomology of the foliated manifold $(M, \mathcal{F})$ satisfies the Poincaré duality (W is the basic manifold of $(M, \mathcal{F})$). Additionally, in compact orientable case for $q = \operatorname{codim} \mathcal{F}$ $H^q(M/\mathcal{F}) = H^q_{A(M,\mathcal{F})}(W) = 0$ or $\mathbb{R}$.

The Poincaré property is a very important one for a transversely orientable Riemannian foliation $\mathcal{F}$ on a compact manifold M as the following conditions are equivalent [E-H], [K-T], [Ma] and [T]:

(1) the basic cohomology $H(M/\mathcal{F})$ satisfies the Poincaré duality;
(2) $H^q(M/\mathcal{F}) \neq 0$;
(3) the foliation $\mathcal{F}$ is minimalizable.

It is well-known that on the total space B of the principal bundle $B(M, SO(q); \mathcal{F})$ of orthonormal frames of the normal bundle of $\mathcal{F}$ there exists a foliation $\mathcal{F}_1$ whose leaves are holonomy coverings of leaves of $\mathcal{F}$.

Moreover, this foliation is transversely parallelizable. Its codimension is equal to $q+q(q-1)/2$ and $H^{q}(M,\mathcal{F})=H^{q+q(q-1)/2}(B,\mathcal{F}_1)$, cf. [E-H-S]. Let $A_{\mathcal{F}_1}$ the Lie algebroid of the foliation $\mathcal{F}_1$. Combining the above results we have the following equivalence:

- *The foliation $\mathcal{F}$ is minimalisable if and only if $A_{\mathcal{F}_1}$ is a TUIO-Lie algebroid.*

The epimorphy of the fibre integral $\int_{(A,\varepsilon)}:\Omega^{*}_{(A,\varepsilon)}(M)\to\Omega^{*-n}(M)$ [K5, Prop.4.2.1(e)] easily gives the following implication [K6] (by assuming the compactness and the orientability of M):

$$A \text{ is a } TUIO\text{-Lie algebroid} \implies H_A^{n+m}(M)\neq 0. \tag{1.2}$$

(Remark: Theorem 1.1 yields more: $H_A^{n+m}(M)=\mathbb{R}$).

Open question. Can the implication in (1.2) be changed by the equivalence?

The paper is dedicated to the study of transitive Lie algebroids with the trivial 1-rank adjoint bundle of isotropy Lie algebras $\boldsymbol{g}\cong M\times\mathbb{R}$. We discover that a locally conformal symplectic structure defines such a Lie algebroid. We prove that this algebroid is a TUIO-Lie algebroid if and only if the associated l.c.s. structure is a globally conformal symplectic structure.

2. Main results

We recall [K2], [K5] that a connection $\lambda:TM\to A$ in a transitive Lie algebroid A determines a covariant derivative ∇ in the adjoint bundle $\boldsymbol{g}$ by the formula: $\nabla_X\nu=[\![\lambda X,\nu]\!]$, $\nu\in\operatorname{Sec}\boldsymbol{g}$. If the structural Lie algebras $\boldsymbol{g}_{|x}$ are abelian, then to all connections λ there corresponds the same covariant derivative ∇ in $\boldsymbol{g}$, called then *canonical* or *characteristic*, and ∇ is flat.

Each flat covariant derivative in $\boldsymbol{g}=M\times\mathbb{R}$ is of the form

$$\nabla_X f=\partial_X f+\omega(X)\cdot f$$

where ω is a closed 1-differentiable form on M. Such one closed forms define new cohomology operators, [G-L] and [H-R1].

According to the general structure theorem [K1], [M], we obtain:

- *Each transitive Lie algebroid on M with a trivial adjoint bundle $\boldsymbol{g}\cong M\times\mathbb{R}$ is isomorphic to*

$$A=(M\times\mathbb{R})\bigoplus TM \tag{2.1}$$

with $\gamma = \mathrm{pr}_2 : (M \times \mathbb{R}) \bigoplus TM \to TM$ *as the anchor and the bracket* $[\![\cdot,\cdot]\!]$ *in* $\operatorname{Sec} A$ *defined via some flat covariant derivative* ∇ *in* $M \times \mathbb{R}$ *and a 2-form* $\Omega \in \Omega^2(M)$ *fulfilling the Bianchi identity* $\nabla\Omega = 0$ *in the following way*

$$[\![(f,X),(g,Y)]\!] = (-\Omega(X,Y) + \nabla_X g - \nabla_Y f, [X,Y]).$$

The flat covariant derivative ∇ *is then characteristic.*

Therefore the condition $\nabla\Omega = 0$ is equivalent to $d\Omega = -\omega\wedge\Omega$. Hence a transitive Lie algebroid with trivial adjoint bundle is determined by the following data: a closed 1-form ω and a 2-form Ω such that $d\Omega = -\omega \wedge \Omega$.

According to [H-R1] the pair $(-\omega,\Omega)$ determinig Lie algebroid (2.1) it is precisely a locally conformal symplectic structure on our manifold provided that the 2-form Ω is non-degenerate. Therefore our transitive algebroid of rank 1 are a natural generalisation of the locally conformal symplectic structure. When the 1-form ω is exact the structure is called globally conformal symplectic. For more information about these structures see [G-L], [H-R1], [H-R2], [V].

Let α be any closed 1-form on M, it defines an operator (see [G-L] and [H-R1])

$$d^\alpha \colon \Omega^*(M) \longrightarrow \Omega^{*+1}(M), \quad \gamma \longmapsto d\gamma + \alpha\wedge\gamma.$$

Oviously $d^\alpha d^\alpha = 0$ and the corresponding cohomology we denote by $H^*_\alpha(M)$.

- *If two forms* α *and* α' *are cohomologous in* $H^1_{DR}(M)$ *then* $H^*_\alpha(M)$ *is isomorphic to* $H^*_{\alpha'}(M)$.

For completeness, we show this briefly. If $\alpha' = \alpha + \frac{d\varphi}{\varphi}$, $\varphi > 0$, then the linear isomorphism on cycles $Z_\alpha \to Z_{\alpha'}, \gamma \mapsto \frac{\gamma}{\varphi}$, transforms d^α-exact forms into $d^{\alpha'}$-exact forms giving an isomorphism on cohomology. Notice that, the inverse homomorphism is given by $Z_{\alpha'} \to Z_\alpha, \gamma' \mapsto \varphi\gamma' = \frac{\gamma'}{1/\varphi}$, and $\alpha = \alpha' - \frac{d\varphi}{\varphi} = \alpha' + \frac{d(1/\varphi)}{1/\varphi}$. In particular if α is an exact form then $H^*_\alpha(M) \cong H^*_{DR}(M)$.

The second fundamental theorem on d^α-cohomology is the following [it was discovered by [G-L], for a short elementary proof see [H-R1]]:

(G-L) *if* α *is not exact (*M *is connected and orientable,* $\dim M = m$, *but* M *can be compact or not) then* $H^m_\alpha(M) = 0$, *i.e. the following differential equation*

$$\Delta = d\gamma + \alpha\wedge\gamma$$

has a global solution $\gamma \in \Omega^{m-1}(M)$ *for each* m*-form* $\Delta \in \Omega^m(M)$.

With this notation in mind, we prove the following theorem:

Theorem 2.1. *Let M be a connected orientable m-manifold. If ∇ is an flat covariant derivative in* $\boldsymbol{g} = M \times \mathbb{R}$ *and* $\nabla_X f = \partial_X f + \omega(X) \cdot f$ *for a closed 1-differential form* $\omega \in \Omega^1(M)$, *then*

(a) *the Lie algebroid A, see (2.1), is a TUIO-Lie algebroid if and only if the form ω is exact, in particular, if $H^1(M) = 0$, then A is a TUIO-Lie algebroid,*
(b) $H_A^{1+m}(M) = H_{-\omega}^m(M)$,
(c) $H_{A,c}^{1+m}(M) = H_{-\omega,c}^m(M)$,

The above theorem permits us to demonstrate our main result:

Theorem 2.2. *Let M be a compact orientable m-manifold and let A be a transitive Lie algebroid with trivial 1-rank adjoint bundle of isotropy Lie algebras in* $\boldsymbol{g} \cong M \times \mathbb{R}$. *Then A is a TUIO-Lie algebroid if and only if* $H_A^{1+m}(M) \neq 0$.

Proof. We have only to demonstrate that the condition $H_A^{1+m}(M) \neq 0$ implies that A is a TUIO-Lie algebroid. If $H_A^{1+m}(M) \neq 0$ Theorem 2.1 ensures that $H_{-\omega}^m(M) \neq 0$. In that case the 1-form $-\omega$ must be exact as for non-exact 1-forms α $H_\alpha^m(M) = 0$, cf. [G-L] and [H-R1]. So according to Theorem 2.1 as the form ω is exact, the Lie algebroid A is a TUIO-Lie algebroid. □

For local conformally symplectic structures we have the following corollary:

Corollary 2.1. *Let (ω, Ω) be locally conformal symplectic structure on M. Then the associated transitive Lie algebroid of rank 1 is a TUIO-algebroid if and only if the locally conformal symplectic structure is a global conformal symplectic structure.*

3. Proof of theorem 2.1

First we have to show that the transitive Lie algebroid (2.1) described above is a TUIO-Lie algebroid if and only if ω is exact. If f is ad_A-invariant, then f is ∇-constant and, by assuming that f is non-singular, we obtain $\omega = d(-\ln(f))$ and, conversely, if $\omega = d(\tilde{f})$, then $f = e^{-\tilde{f}}$ is a non-singular ad_A-invariant cross-section of $\boldsymbol{g} = M \times \mathbb{R}$.

Denote by $\mathrm{pr}_1 : (M \times \mathbb{R}) \bigoplus TM \to M \times \mathbb{R}$ the projection onto the first factor. Let d_A be the exterior derivative of A-differential forms $\Omega_A(M) = \mathrm{Sec} \bigwedge A^\star$. For a homomorphism of Lie algebroids $F : A' \to A$ over $t : M' \to M$ the pullback of differential forms $F^\star : \Omega_A(M) \to \Omega_{A'}(M')$ is defined by $(F^\star\Psi)_{|x}(v_1, ..., v_k) = \Psi_{|tx}(Fv_1, ..., Fv_k)$ and the following equalities $F^\star(\Psi \wedge \Phi) = F^\star\Psi \wedge F^\star\Phi$, $F^\star(d_A\Psi) = d_{A'}(\Psi)$ hold.

Represent the algebra of A-differential forms for $A = (M \times \mathbb{R}) \bigoplus TM$ as the skew tensor product of the anticommutative graded algebras

$$\begin{aligned} \Omega_A(M) &= \operatorname{Sec}\left(\bigwedge (M \times \mathbb{R})^\star \bigotimes \bigwedge T^\star M\right) \\ &= \bigwedge \mathbb{R}^\star \bigotimes \Omega(M) \end{aligned}$$

where $\bigwedge \mathbb{R}^\star$ is the exterior algebra over a one-dimensional graded vector space (homogeneous of degree 1). Therefore each $(k+1)$-form $\Phi^{k+1} \in \Omega_A^{k+1}(M)$ has the unique representation of the form

$$\Phi^{k+1} = 1 \otimes \varphi^{k+1} + \mathbf{1}^\star \otimes \varphi^k \tag{3.1}$$

where φ^k and φ^{k+1} are differential forms on M of degrees k and $k+1$, respectively, and $\mathbf{1}^\star = \mathrm{id}_{\mathbb{R}}$. Clearly, $1 \otimes \varphi^{k+1} = \gamma^\star \varphi^{k+1}$ and $\mathbf{1}^\star \otimes \varphi^k = \mathrm{pr}_1^\star(\varepsilon^\star) \wedge \gamma^\star \varphi^k$ for the cross-section $\varepsilon^\star$ of $M \times \mathbb{R}^\star$ defined by $\varepsilon^\star(x) \equiv \mathbf{1}^\star$. The structure of the skew tensor product of the anticommutative graded algebras $\bigwedge \mathbb{R}^\star$ and $\Omega(M)$ is given by

$$\begin{aligned} &\left(1 \otimes \varphi^{k+1} + \mathbf{1}^\star \otimes \varphi^k\right) \wedge \left(1 \otimes \psi^{l+1} + \mathbf{1}^\star \otimes \psi^l\right) \\ = \; & 1 \otimes \left(\varphi^{k+1} \wedge \psi^{l+1}\right) + \mathbf{1}^\star \otimes \left(\varphi^k \wedge \psi^{l+1} + (-1)^{k+1} \varphi^{k+1} \wedge \psi^l\right). \end{aligned}$$

Via representation (3.1) we easily show that the differential d_A in $\bigwedge \mathbb{R}^\star \bigotimes \Omega(M)$ is given by

$$\begin{aligned} &d_A\left(1 \otimes \varphi^k + \mathbf{1}^\star \otimes \varphi^{k-1}\right) \\ = \; & 1 \otimes \left(d\varphi^k + \Omega \wedge \varphi^{k-1}\right) + \mathbf{1}^\star \otimes \left(\omega \wedge \varphi^{k-1} - d\varphi^{k-1}\right). \end{aligned}$$

To see this, we have, at first,

$$d_A\left(1 \otimes \varphi^k\right) = d_A\left(\gamma^\star \varphi^k\right) = \gamma^\star\left(d\varphi^k\right) = 1 \otimes d\varphi^k$$

and

$$d_A\left(\mathbf{1}^\star \otimes 1\right) = d_A\left(\mathrm{pr}_1^\star(\varepsilon^\star)\right) = 1 \otimes \Omega + \mathbf{1}^\star \otimes \omega;$$

indeed,

$$\begin{aligned} & d_A\left(\mathrm{pr}_1^\star(\varepsilon^\star)\right)((f,X),(g,Y)) \\ = \; & \partial_X g - \partial_Y f - \mathrm{pr}_1^\star(\varepsilon^\star)(\llbracket (f,X),(g,Y) \rrbracket) \\ = \; & \partial_X g - \partial_Y f - \mathrm{pr}_1^\star(\varepsilon^\star)(-\Omega(X,Y) + \nabla_X g - \nabla_Y f, [X,Y]) \\ = \; & \partial_X g - \partial_Y f - (-\Omega(X,Y) + \nabla_X g - \nabla_Y f) \\ = \; & \partial_X g - \partial_Y f + \Omega(X,Y) - (\partial_X g + \omega(X) \cdot g) + \partial_Y f + \omega(Y) \cdot f \\ = \; & \Omega(X,Y) - \omega(X) \cdot g + \omega(Y) \cdot f \\ = \; & \Omega(X,Y) + \mathrm{pr}_1^\star(\varepsilon^\star) \wedge \mathrm{pr}_1^\star(\omega)((f,X),(g,Y)) \\ = \; & (1 \otimes \Omega + \mathbf{1}^\star \otimes \omega)((f,X),(g,Y)). \end{aligned}$$

So

$$\begin{aligned}
& d_A\left(1\otimes\varphi^k+\mathbf{1}^\star\otimes\varphi^{k-1}\right)\\
= \; & d_A\left(1\otimes\varphi^k\right)+d_A\left(\left(\mathbf{1}^\star\otimes 1\right)\wedge\left(1\otimes\varphi^{k-1}\right)\right)\\
= \; & 1\otimes d\varphi^k+d_A\left(\mathbf{1}^\star\otimes 1\right)\wedge\left(1\otimes\varphi^{k-1}\right)-\left(\mathbf{1}^\star\otimes 1\right)\wedge d_A\left(1\otimes\varphi^{k-1}\right)\\
= \; & 1\otimes d\varphi^k+\left(1\otimes\Omega+\mathbf{1}^\star\otimes\omega\right)\wedge\left(1\otimes\varphi^{k-1}\right)-\left(\mathbf{1}^\star\otimes 1\right)\wedge\left(1\otimes d\varphi^{k-1}\right)\\
= \; & 1\otimes\left(d\varphi^k+\Omega\wedge\varphi^{k-1}\right)+\mathbf{1}^\star\otimes\left(\omega\wedge\varphi^{k-1}\right)-\mathbf{1}^\star\otimes d\varphi^{k-1}\\
= \; & 1\otimes\left(d\varphi^k+\Omega\wedge\varphi^{k-1}\right)+\mathbf{1}^\star\otimes\left(\omega\wedge\varphi^{k-1}-d\varphi^{k-1}\right).
\end{aligned}$$

We look at the graded algebra $\bigwedge\mathbb{R}^\star\bigotimes\Omega(M)$ as follows:

$$\begin{aligned}
\bigwedge\mathbb{R}^\star\bigotimes\Omega(M) &= \left(\mathbb{R}\bigotimes\Omega^\bigstar(M)\right)\bigoplus\left(\mathbb{R}^\star\bigotimes\Omega^{\bigstar-1}(M)\right)\\
&\equiv \Omega^\bigstar(M)\bigoplus\Omega^{\bigstar-1}(M)\\
1\otimes\varphi^k+\mathbf{1}^\star\otimes\varphi^{k-1} &\longmapsto \left(\varphi^k,\varphi^{k-1}\right),
\end{aligned}$$

$$\left(\varphi^k,\varphi^{k-1}\right)\wedge\left(\psi^k,\psi^{k-1}\right)=\left(\varphi^{k+1}\wedge\psi^{l+1},\varphi^k\wedge\psi^{l+1}+(-1)^{k+1}\varphi^{k+1}\wedge\psi^l\right),$$

Therefore the exterior derivative on the level of k-forms is then given by the formula

$$d_A^k:\Omega^k(M)\bigoplus\Omega^{k-1}(M)\longrightarrow\Omega^{k+1}(M)\bigoplus\Omega^k(M)$$

$$d_A\left(\varphi^k,\varphi^{k-1}\right)=\left(d\varphi^k+\Omega\wedge\varphi^{k-1},\omega\wedge\varphi^{k-1}-d\varphi^{k-1}\right).$$

In particular, $d_A\left(\varphi^k,\varphi^{k-1}\right)=0$ if and only if $d\varphi^k=-\Omega\wedge\varphi^{k-1}$ and $d\varphi^{k-1}=\omega\wedge\varphi^{k-1}$.

Let $m=\dim M$. The maximal degree of A-differential forms is $m+1$,

$$\Omega_A^{m+1}\equiv\Omega^{m+1}(M)\bigoplus\Omega^m(M)=0\bigoplus\Omega^m(M)=\Omega^m(M)$$

and

$$\begin{aligned}
d_A^m:\Omega_A^m(M)\equiv\Omega^m(M)\oplus\Omega^{m-1}(M) &\longrightarrow \Omega^m(M)\equiv\Omega_A^{m+1}(M),\\
\left(\varphi^m,\varphi^{m-1}\right) &\longmapsto \omega\wedge\varphi^{m-1}-d\varphi^{m-1}.
\end{aligned}$$

Put

$$\bar{d}^{k-1}:\Omega^{k-1}(M)\longrightarrow\Omega^k(M),\ \varphi^{k-1}\longmapsto\omega\wedge\varphi^{k-1}-d\varphi^{k-1}.$$

Clearly $\bar{d}=-d^{-\omega}$ and $\bar{d}\circ\bar{d}=0$,

$$H_A^{m+1}(M)=H^m\left(\Omega(M),\bar{d}\right)=H_{-\omega}^m\left(\Omega(M),\bar{d}\right),$$

and $H_A^{m+1}(M)$ does not depend on the 2-form Ω.

Analogously

$$H_{A,c}^{m+1}(M) = H_c^m\left(\Omega(M), \bar{d}\right) = H_{-\omega,c}^m\left(\Omega(M), \bar{d}\right),$$

which ends the proof.

Therefore $H_A^{m+1}(M) = 0$ if and only if, for each m-form $\Delta \in \Omega^m(M)$, there exists an $m-1$-form $\varphi \in \Omega^{m-1}(M)$ such that $\Delta = \omega \wedge \varphi - d\varphi$.

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