Proposition 1.43. *(i) Let $u, v \in W_2^1(\Omega) \cap L_\infty(\Omega)$. Then $uv \in W_2^1(\Omega) \cap L_\infty(\Omega)$ with*

$$\partial_j(uv) = \partial_j uv + u\partial_j v, \qquad i = 1, \dots, n \tag{1.56}$$

(ii) *Let Ω, Ω_1 be two open sets in $\mathbb{R}^n$ and let $H : \Omega_1 \to \Omega$ be a $C^1(\bar{\Omega})$ diffeomorphism. If $u \in W_2^1(\Omega)$ then $u \circ H \in W_2^1(\Omega')$ and*

$$\int_{\Omega_1} (u \circ H)\partial_j \phi d\mathbf{y} = -\int_{\Omega_1} \sum_{i=1}^{n} (\partial_i u \circ H))\partial_j H_i \phi d\mathbf{y} \tag{1.57}$$

Proof. Using Friedrichs lemma, we find sequences $(u_k)_{k\in\mathbb{N}}$, $(v_k)_{k\in\mathbb{N}}$ in $C_0^\infty(\Omega)$ such that

$$u_k \to u, \qquad v_k \to v$$

in $L_2(\Omega)$ and for any $\Omega' \Subset \Omega$ we have

$$\nabla u_k \to \nabla u, \qquad \nabla v_k \to \nabla v$$

in $L_2(\Omega')$. Moreover, from the construction of the mollifiers we get

$$\|u_k\|_{L_\infty(\Omega)} \le \|u\|_{L_\infty(\Omega)} \qquad \|v_k\|_{L_\infty(\Omega)} \le \|v\|_{L_\infty(\Omega)}.$$

On the other hand $u_k = \int_{\mathbb{R}^n} u(x-y)\omega_{1/k}(y)\,dy$

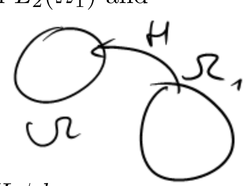
$$\int_{\Omega} u_k v_k \partial_j \phi d\mathbf{x} = -\int_{\Omega} (\partial_j u_k v_k + u_k \partial_j v_k)\phi d\mathbf{x}$$

for any $\phi \in C_0^\infty(\Omega)$. Thanks to the compact support of ϕ, the integration actually occurs over compact subsets of Ω and we can use L_2 convergence of $\nabla u_k, \nabla v_k$. Thus

$$\int_{\Omega} uv\partial_j \phi d\mathbf{x} = -\int_{\Omega} (\partial_j uv + u\partial_j v)\phi d\mathbf{x}$$

and the fact that $uv \in W_2^1(\Omega)$ follows from $\partial_j u, \partial_j v \in W_2^1(\Omega)$ and $u, v \in L_\infty(\Omega)$. The proof of the second statement follows similarly. We select sequence $(u_k)_{k\in\mathbb{N}}$ as above; then clearly $u_k \circ H \to u \circ H$ in $L_2(\Omega_1)$ and

$$(\partial_i u_k \circ H)\partial_j H_i \to (\partial_i u \circ H)\partial_j H_i$$



in $L_2(\Omega_1')$ for any $\Omega_1' \Subset \Omega$. For any $\psi \in C_0^\infty(\Omega_1)$ we get

$$\int_{\Omega_1} (u_k \circ H)\partial_j \phi d\mathbf{y} = -\int_{\Omega_1} \sum_{i=1}^{k} (\partial_i u_k \circ H)\partial_j H_i \phi d\mathbf{y}$$

and in the limit we obtain (1.57).

The next result shows that elements from $W_2^1(\Omega)$ can by, in L_2 norm, approximated by finite differences.

Proposition 1.44. *The following properties are equivalent:* $\| \ \|_{0,\Omega'} = \| \ \|_{L_2(\Omega')}$

(i) $u \in W_2^1(\Omega)$,
(ii) there is C *such that for any* $\phi \in C_0^\infty(\Omega)$ *and* $i = 1, \ldots, n$

$u \in W_2^1(\Omega)$

$\exists g \ \int_\Omega u\partial_i \varphi = -\int_\Omega g\varphi$

$$\int_\Omega u\partial_i\phi d\mathbf{x} \le C\|\phi\|_0, \tag{1.58}$$

(iii) there is a constant C *such that for any* $\Omega' \Subset \Omega$ *and all* $\mathbf{h} \in \mathbb{R}^n$ *with* $|\mathbf{h}| \le \operatorname{dist}(\Omega', \partial\Omega)$ *we have*

$$\|\tau_h u - u\|_{0,\Omega'} \le C|\mathbf{h}|, \tag{1.59}$$

where $(\tau_h u)(\mathbf{x}) = u(\mathbf{x} + \mathbf{h})$. *In particular, if* $\Omega = \mathbb{R}^n$, *then*

$$\|\tau_h u - u\|_0 \le \cancel{C}|\mathbf{h}|\|\nabla u\|_0. \tag{1.60}$$

Proof. $(i) \Rightarrow (ii)$ follows from the definition.
$(ii) \Rightarrow (i)$. Eqn. (1.58) shows that

$$\phi \to \int_\Omega u\partial_i\phi d\mathbf{x},$$

$\ell(\varphi)$ $\varphi \in L_2(\Omega)$

extends to a bounded functional on $L_2(\Omega)$ and thus there is $v_i \in L_2(\Omega)$ such that

$\forall \ell(\varphi) = (\varphi, v_i)$

$$\int_\Omega u\partial_i\phi d\mathbf{x} = -\int_\Omega v_i\phi d\mathbf{x},$$

$\varphi \in L_2(\Omega)$

for any $\phi \in C_0^\infty(\Omega)$.
$(i) \Rightarrow (iii)$. Let us take $u \in C_0^\infty(\mathbb{R}^n)$. For $\mathbf{x}, \mathbf{h} \in \mathbb{R}^n$ and $t \in \mathbb{R}$ we define

$$v(t) = u(\mathbf{x} + t\mathbf{h}).$$

Then $v'(t) = \mathbf{h}(\nabla u)(\mathbf{x} + t\mathbf{h})$ and

$$u(\mathbf{x} + \mathbf{h}) - u(\mathbf{x}) = v(1) - v(0) = \int_0^1 \mathbf{h}\nabla u(\mathbf{x} + t\mathbf{h})dt.$$

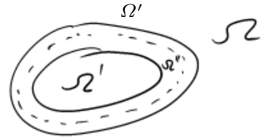
$$\le \int_0^1 |h|^2 dt \cdot \int_0^1 |Du(x+th)|^2 dt$$

Hence

$$|\tau_h u(\mathbf{x}) - u(\mathbf{x})|^2 \le |\mathbf{h}|^2 \int_0^1 |\nabla u(\mathbf{x} + t\mathbf{h})|^2 dt$$

so that

$$\int_{\Omega'} |\tau_h u(\mathbf{x}) - u(\mathbf{x})|^2 d\mathbf{x} \le |\mathbf{h}|^2 \int_0^1 \left(\int_{\Omega'} |\nabla u(x + t\mathbf{h})|^2 d\mathbf{x} \right) dt$$



$$= |\mathbf{h}|^2 \int_0^1 \left(\int_{\Omega' + t\mathbf{h}} |\nabla u(\mathbf{y})|^2 d\mathbf{y} \right) dt.$$

If $|\mathbf{h}| < \mathrm{dist}(\Omega', \partial\Omega)$, then there is Ω'' such that $\Omega' + t\mathbf{h} \subset \Omega'' \Subset \Omega$ for all $t \in [0, 1]$ and thus

$$\int_{\Omega'} |\tau_h u(\mathbf{x}) - u(\mathbf{x})|^2 d\mathbf{x} \le |\mathbf{h}|^2 \int_{\Omega''} |\nabla u(\mathbf{y})|^2 d\mathbf{x} \qquad \le |h| \int_\Omega |Du(y)|^2 dy$$

which gives (1.59) for $u \in C_0^\infty(\mathbb{R}^n)$. Let $u \in W_2^1(\Omega)$. Then, by the Friedrichs lemma, we find $(u_k)_{k\in\mathbb{N}}$, $u_k \in C_0^\infty(\mathbb{R}^n)$ such that $u_k \to u$ in $L_2(\Omega)$ and $\nabla u_k \to \nabla u$ in $L_2(\Omega')$ for any $\Omega' \Subset \Omega$. Noting that $\tau_h u_k \to \tau_h u$ in $L_2(\Omega')$ we can pass to the limit above, obtaining,

$$\|\tau_h u - u\|_{0,\Omega'} \le |\mathbf{h}| \sqrt{\int_{\Omega''} |\nabla u(\mathbf{y})|^2 d\mathbf{x}} \le C|\mathbf{h}|.$$

If $\Omega = \mathbb{R}^n$, then in all calculations above we can replace Ω', Ω'' by $\mathbb{R}^n$.

$(iii) \Rightarrow (ii)$. If (1.59) holds then, taking $\Omega' \Subset \Omega$, $\phi \in C_0^\infty(\Omega)$ with $\mathrm{supp}\phi \subset \Omega'$ and $|\mathbf{h}| < \mathrm{dist}(\Omega', \partial\Omega)$, we obtain

$$\left| \int_\Omega (\tau_h u - u)\phi d\mathbf{x} \right| \le C|\mathbf{h}|\|\phi\|_0.$$

On the other hand

$$\int_\Omega (\tau_h u - u)(\mathbf{x})\phi(\mathbf{x}) d\mathbf{x} = \int_\Omega u(\mathbf{y})(\tau_{-h}\phi - \phi)(\mathbf{y}) d\mathbf{y},$$

$$\int_\Omega u \varphi_{x_i} dy \le C\|\varphi\|_0$$

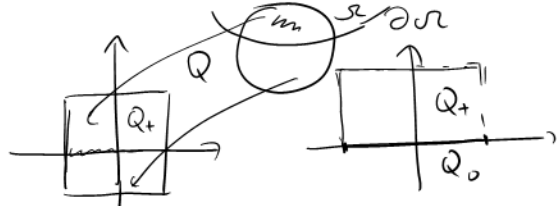
so

$$\left| \int_\Omega u \frac{(\tau_{-h}\phi - \phi)}{|\mathbf{h}|} d\mathbf{y} \right| \le C\|\phi\|_0.$$

Choosing $\mathbf{h} = t\mathbf{e}_i, i = 1, \dots, n$ and passing to the limit with $t \to 0$, we obtain (1.58).

1.3.8 Localization and flattening of the boundary

Assume that Ω is an open, bounded set with boundary $\partial\Omega$ which is an $n-1$ dimensional C^m manifold; further assume that that Ω lies locally at one side



of the boundary. Denote $Q = \{\mathbf{y} \in \mathbb{R}^n;\ |y_i| < 1, i = 1, \ldots, n\}$, $Q_0 = \{\mathbf{y} \in Q;\ y_n = 0\}$ and $Q_+ = \{\mathbf{y} \in Q;\ x_n > 0\}$. Then we have a finite local atlas on $\partial\Omega$, that is, a finite collection $\{B_j, H_j\}_{1 \le j \le N}$ where B_j are open balls covering $\partial\Omega$, $H_j : Q \to B_j$ are C^m diffeomorphisms with positive Jacobians which are bijections of Q, Q_0 and Q_+ onto $B_j, B_j \cap \partial\Omega$ and $B_j \cap \Omega$, respectively.

Given the local atlas $\{B_j, H_j\}_{1 \le j \le N}$, we construct a finite open subcover $\{G_j\}_{1 \le j \le N}$ in such a way that $G_j \Subset B_j$ and $\partial\Omega \subset \bigcup_{j=1}^N G_j$. In fact, we can take $G_j = B_j^k$ to be balls concentric with B_j and slightly smaller radius, say, $\rho_j^k = r_j - 1/k$ for some k. Indeed, suppose it is impossible, then for any k there is $x_k \in \partial\Omega$ such that $x_k \notin \bigcup_{j=1}^N B_j^k$. From compactness of $\partial\Omega$ we obtain an accumulation point $x \in \partial\Omega$. Hence $x \in B_j$ for some j and thus $x \in B_j^k$ for sufficiently large k. This contradicts the construction that x is an accumulation point of points which are outside $\bigcup_{j=1}^N B_j^k$. Defining $G_0 = \Omega \setminus \bigcup_{j=1}^N \bar{G}_j$ we further get an open set G_0 with $\bar{G}_0 \subset \Omega$. Thus

$G_0 \subset\subset \Omega$

$$\bar{\Omega} \subset \Omega \cup \bigcup_{j=1}^N G_j, \qquad \Omega \subset \bigcup_{j=0}^N \bar{G}_j.$$

Now, we choose $\alpha_j \in C_0^\infty(\mathbb{R}^n)$ satisfying $0 \le \alpha \le 1$, supp$\alpha_j \subset B_j$ and $\alpha_j = 1$ on $\bar{G}_j$. Further, $\alpha \in C_0^\infty(\mathbb{R}^n)$ satisfies

$$\mathrm{supp}\alpha \subset \Omega \cup \bigcup_{j=1}^N G_j, \quad 0 \le \alpha \le 1, \quad \alpha = 1 \text{ on } \bar{\Omega}.$$

Then define

$$\beta_j(\mathbf{x}) = \frac{\alpha(\mathbf{x})\alpha_j(\mathbf{x})}{\sum_{k=0}^N \alpha_k(\mathbf{x})}$$

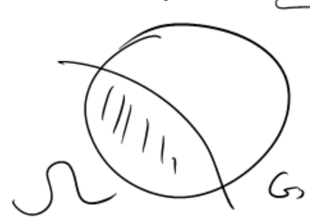
for $\mathbf{x} \in \bigcup_{j=0}^N \bar{G}_j$ and $\beta_j(\mathbf{x}) = 0$ for $\mathbf{x} \in \mathbb{R}^n \setminus \bigcup_{j=0}^N \bar{G}_j$. We note that each β_j is well defined. Indeed, at least one $\alpha_j(\mathbf{x})$ is equal 1 on $\bigcup_{j=0}^N \bar{G}_j$ so that the denominator is at least 1 there. On the other hand, α vanishes outside a compact set contained in $\bigcup_{j=0}^N G_j$. Hence, $\beta_j \in C_0^\infty(\mathbb{R}^n)$, supp$\beta_j \subset B_j$, $\beta_j \ge 0$ and

$$\sum_{j=0}^N \beta_j(\mathbf{x}) = 1$$

for $\mathbf{x} \in \bar{\Omega}$.

We call the collection $\{\beta_j\}_{j=0}^N$ a partition of unity subordinated to the open cover $\{G_j\}_{j=0}^N$ of Ω and $\{\beta_j\}_{j=1}^N$ a partition of unity subordinated to the open cover $\{G_j\}_{j=1}^N$ of Ω of $\partial\Omega$.

Suppose now we have $u \in W_2^1(\Omega)$. Then $u = \sum_{j=0}^N \beta_j u$ on Ω and, by Proposition 1.43 (i), $\beta_j u \in W_2^1(\Omega \cap G_j)$, $j = 1, \ldots, N$. Using Proposition 1.43



(ii) we see that for each $j = 1, \dots, N$ we $(\beta_j u) \circ H_j \in W_2^1(Q_+)$ with support in Q. Define $\Lambda : W_2^1(\Omega) \to \mathring{W}{}_1^2(Q) \times [W_2^1(\Omega)]^N$ by

$$\Lambda u = (\beta_0 u, \beta_1 u \circ H_1, \dots, \beta_N u \circ H_N).$$

Note that we can write $\beta_0 u \in \mathring{W}{}_1^2(Q)$ as $\beta_0 u$ has compact support in Ω and thus, by Friedrichs lemma, it can be approximated by $C_0^\infty(Q)$ functions. The mapping Λ is a linear injection as if $u(x) \neq 0$, then at least one entry of Λ must be nonzero as βs sum up to 1. Also, using Proposition 1.43, we can show that the norm on $\Lambda W_2^1(\Omega)$ is equivalent to the norm on $W_2^1(\Omega)$ and thus Λ is an isomorphism of $W_2^1(\Omega)$ onto its closed image.

$u_n \to u$ in $W^1_2(\Omega)$

1.3.9 Extension operator

We observed that one of the main obstacles in proving that $W_2^1(\Omega)$ can be obtained by closure of restrictions of $C_0^\infty(\mathbb{R}^n)$ functions to Ω is that we have no control over the regularization at points close to the boundary of Ω. A remedy could be if we are able to show that any function $W_2^1(\Omega)$ can be extended to a function from $W_2^1(\Omega)$.

Indeed, we have

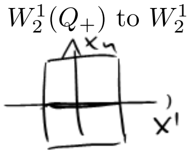
Theorem 1.45. *Suppose that Ω is bounded with a C^1 boundary $\partial\Omega$. Then there exists a linear extension operator*

$$E : W_2^1(\Omega) \to W_2^1(\mathbb{R}^n)$$

such that for any $u \in W_2^1(\Omega)$

1. $Eu|_\Omega = u$;
2. $\|Eu\|_{0,\mathbb{R}^n} \le C\|u\|_{0,\Omega}$;
3. $\|Eu\|_{1,\mathbb{R}^n} \le C\|u\|_{1,\Omega}$;

Proof. We begin by showing that we can construct an extension operator from $W_2^1(Q_+)$ to $W_2^1(Q)$. Let $u \in W_2^1(Q_+)$ and define extension by reflection

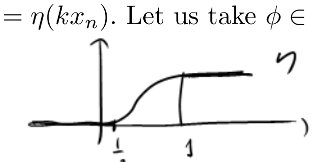


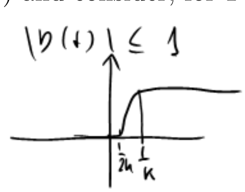
$$u^*(\mathbf{x}', x_n) = \begin{cases} u(\mathbf{x}', x_n) & \text{for } x_n > 0, \\ u(\mathbf{x}', -x_n) & \text{for } x_n < 0 \end{cases}$$

where $\mathbf{x}' = (x_1, \dots, x_{n-1})$. In the same way, we define the odd reflection

$$\tilde{u}(\mathbf{x}', x_n) = \begin{cases} u(\mathbf{x}', x_n) & \text{for } x_n > 0, \\ -u(\mathbf{x}', -x_n) & \text{for } x_n < 0 \end{cases}$$

Further, we define a cut-off function close to $x_n = 0$, that is, we take a $C^\infty(\mathbb{R})$ function η which satisfies $\eta(t) = 1$ for $t \ge 1$ and $\eta(t) = 0$ for $t \le 1/2$ and define $\eta_k(x_n) = \eta(kx_n)$. Let us take $\phi \in C_0^\infty(Q)$ and consider, for $1 \le i \le n-1$,





$1 \le i \le n-1$

$$\int_Q u^* \partial_{x_i}\varphi\, d\underline{x} = \int_{Q^+} u\, \partial_{x_i}\varphi\, dx + \int_{Q^-} u^* \partial_{x_i}\varphi\, d\underline{x}$$

$$= \quad - \,''\, - \quad \int_{Q_0}\int_{-1}^{0} u(x',-x_n)\,\partial_{x_i}\varphi(x',x_n)\, dx \qquad x_n \to -x$$

$$\int_Q u^*\partial_{x_i}\phi d\mathbf{x} = \int_{Q_+} u\partial_{x_i}\psi d\mathbf{x} \qquad + \int_{Q_0}\int_0 u(x',x_n)\,\partial_{x_i}\varphi(x',-x_n)\, dx$$

where $\psi(\mathbf{x}', x_n) = \phi(\mathbf{x}', x_n) + \phi(\mathbf{x}', -x_n)$. Typically, ψ is not zero at Q_0 and cannot be used as a test function. However, $\eta_k(x_n)\psi(\mathbf{x}) \in C_0^\infty(Q_+)$ and we can write

$$\int_{Q_+} u\partial_{x_i}(\eta_k\psi)d\mathbf{x} = -\int_{Q_+} \partial_{x_i} u\eta_k\psi d\mathbf{x}.$$

However, $\partial_{x_i}\eta_k\psi = \eta_k\partial_{x_i}\psi$ as η does not depend on x_i, $i = 1, \ldots, n-1$ and hence

$$\int_{Q_+} \eta_k u\partial_{x_i}\psi d\mathbf{x} = -\int_{Q_+} \partial_{x_i} u\eta_k\psi d\mathbf{x}.$$

We can pass to the limit by dominated convergence getting

$$\int_{Q_+} u\partial_{x_i}\psi d\mathbf{x} = -\int_{Q_+} \partial_{x_i} u\psi d\mathbf{x},$$

$$-\int_{Q_+}\partial_{x_i} u\, \varphi(x',x_n)\, dx \;\; \| \;\; -\int_{Q_0}\int_0 \partial_{x_i} u\, \varphi(x',-x_n)\, dx_n\, dx' \qquad -x_n \to x_n$$

so that, returning to Q

$$- \int_{Q_+} \partial_{x_i} u\,(\varphi(x',x_n) + \varphi(x',-x_n))\, dx \;\; ''$$

$$\int_Q u^*\partial_{x_i}\phi d\mathbf{x} = -\int_{Q_+} \partial_{x_i} u\psi d\mathbf{x} = -\int_Q (\partial_{x_i} u)^*\phi d\mathbf{x}. \qquad -\int_{Q_-}(\partial_{x_i} u)(x',-x_n)\,\varphi(x)\, dx$$

Now let us consider differentiability with respect to x_n. Again, taking $\phi \in C_0^\infty(Q)$ $\quad Q_- \quad x_n \to x_n$

$$\int_Q u^*\partial_{x_n}\phi d\mathbf{x} = \int_{Q_+} u\partial_{x_n}\chi d\mathbf{x} \qquad -\partial_{x_n}\phi(x,-x_n)$$

where $\chi(\mathbf{x}', x_n) = \phi(\mathbf{x}', x_n) - \phi(\mathbf{x}', -x_n)$. If we again use η_k, then

$$\partial_{x_n}\eta_k\chi = \eta_k\partial_{x_n}\chi + \chi\partial_{x_n}\eta_k$$

where $\partial_{x_n}\eta_k(x_n) = k\eta'(kx_n)$. Then

$$\int_{Q_0}\int_0^{1/k} |u(x)|\, dx_n\, dx' \to 0_{k\to\infty}$$

$$k\left|\int_{Q_+} u(\mathbf{x})\eta'(kx_n)\chi(\mathbf{x})dx\right| \le \cancel{k}CM\int_{Q_0}\left(\int_0^{1/k} |u(\mathbf{x})\cancel{x_n}|dx_n\right)d\mathbf{x}' \le \cancel{CM\int_{Q_+} |u(\mathbf{x})|d\mathbf{x}} \to 0$$

as $k \to \infty$, where $C = \sup_{t\in[0,1]} |\eta'(t)|$ and M is obtained from the estimate

$$|\chi(\mathbf{x}', x_n) \le M|x_n|$$

on Q. Thus

$$\int_{Q_+} u(\partial_{x_n}\eta_k\chi)\, d\underline{x} = \int_{Q_+} u(\partial_{x_n}\eta_k)\chi\, dx + \int_{Q_+} u\eta_k\partial_{x_n}\chi\, dx$$

$$\left|\int_{Q_+} u(\partial_{x_n}\eta_k)\chi\, dx\right| \le k\int_{Q_+} |u|\,|\eta'(kx_n)|\,|\chi(x)|\, dx \qquad \le M|x_n|$$

$$= kC\int_{Q_0} dx'\int_0^{1/k} |u| \qquad |\chi(x',x_n)|\, dx_n$$

$\chi(x',0) = 0$
$\chi(x',x_n) - 0$
$\le M|x_n|$

$\eta(t) = 1 \quad t > 1 \qquad \eta' = 0 \quad t > 1 \qquad \eta'(kx_n) = 0$ the $x_n > \frac{1}{k}$

$$\int_{Q_+} u\partial_{x_n}(\eta_k\chi)d\mathbf{x} = \int_{Q_+} u(\eta_k\partial_{x_n}\chi + \underbrace{\chi\partial_{x_n}\eta_k})d\mathbf{x} \to \int_{Q_+} u\eta_k\partial_{x_n}\chi \quad .$$

and thus we obtain in the limit

$$\int_{Q_+} u\partial_{x_n}\chi d\mathbf{x} = -\int_{Q_+} \partial_{x_n}u\chi d\mathbf{x}.$$

Returning to Q, we obtain

$$\int_Q u^*\partial_{x_n}\phi d\mathbf{x} = \int_{Q_+} u\partial_{x_n}\phi d\mathbf{x} = \int_Q \widetilde{(\partial_{x_n}u)}\phi d\mathbf{x}.$$

We also obtain estimates

$$\|u^*\|_{0,Q} \le 2\|u\|_{0,Q_+} \qquad \|u^*\|_{1,Q} \le 2\|u\|_{1,Q_+}.$$

$u \in W_2^1(\Omega)$ Ω - ohraničená

$G_0 \subset\subset \Omega$ G_j $\partial\Omega \subset \bigcup_j G_j \subset \bigcup_j B_j$

$\overline{G}_j \subset B_j$

β_j supp $\beta_j \subset B_j$ $\sum' \beta_j = 1$ na $\overline{\Omega}$

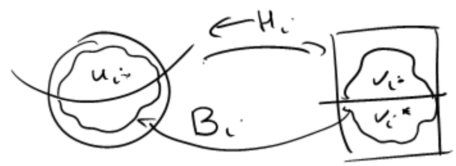
$u = \sum' \beta_i u = \sum' u_i$ $u_0 \in \mathring{W}_2^1(\Omega)$

$u_i \in W_2^1(\Omega \cap B_i)$

Rozšíření u_0 jest [illegible]

$\bar{u} = \begin{cases} u_0 & x \in G_0 \\ 0 & x \notin G_0 \end{cases}$ $\bar{u} \in W_2^1(\mathbb{R}^n)$

$v_i(y) = u_i(H_i(y))$



$v_i \in W_2^1(Q_+)$

$v_i^* \in W_2^1(Q)$

$w_i = v_i^*(H_i^{-1})$

$w_i|_{B_i \cap \Omega} = u_i$ $\|u_i\|_{W_2^1(B_i)} \le C\|u_i\|_{W_2^1(B_i \cap \Omega)}$

Rozšíření definujeme

$Eu = u_0^* + \sum_{i=1}' w_i$

$Eu \in W_2^1(\mathbb{R}^n)$ Eu zřejmě rozšíření

$x \in \overline{\Omega}$, jestli $x \in B_j \Rightarrow w_i = u_i$ $i \ge 1$

$(Eu)(x) = \beta_0 u + \sum' u_i = \beta_0 u + \sum_i' \beta_i u$ $\bar{u}_0 = \beta_0 u$ $i = 0$

$= u(x)$