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## Notation

$\mathbb{N}$	$:= \{0, 1, 2, \dots\}$ is the set of all natural numbers
$\mathbb{N}^*$	$:= \mathbb{N} - \{0\}$
$\mathbb{R}$	is the set of all real numbers
$\mathbb{C}$	is the set of all complex numbers
$\mathbb{R}_+$	$:= [0, +\infty)$ is the set of all nonnegative real numbers
$\mathbb{R}_+^*$	$:= \mathbb{R}_+ - \{0\} = (0, +\infty)$ is the set of all positive real numbers
$\mathbb{R}^n$	$:= \mathbb{R} \times \dots \times \mathbb{R}$ ( $n$ times) is the $n$ -dimensional Euclidean space
$\mathbb{K}$	$:= \mathbb{R}_+^n = \mathbb{R}_+ \times \dots \times \mathbb{R}_+$ is the positive cone of the $n$ -dimensional Euclidean space
$\mathbb{R}_+^{n*}$	$:= \mathbb{R}_+^* \times \dots \times \mathbb{R}_+^*$ ( $n$ times) $= \overset{\circ}{\mathbb{K}}$
$\ \cdot\ $	denotes in general the Euclidean norm in $\mathbb{R}^n$ , but it can also denote the norm in an arbitrary normed vector space (depending upon the context)
$B_\rho(z_0)$	$:= \{z \in E \mid \ z - z_0\  < \rho\}$ denotes the open ball with center $z_0 \in E$ and radius $\rho > 0$ in a normed vector space $E$
$A^T$	denotes the transpose of the matrix $A$

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