Geometric aspects of invariants of finite type of knots in S^3

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The knot invariants of finite type (or Vassiliev invariants) give a coherent and systematic way to study the polynomial invariants of knots in S^3. However the geometric nature and geometric understanding of both the polynomial and the Vassiliev invariants remain missing. That is, it is unclear what kind of geometric knot information is carried by these invariants. In the present talk, we briefly review the well-known facts

on this subject and report some new results.

The two knots are called n-equivalent, where $n \geq 1$, if they cannot be distinguished by the Vasssiliev invariants of order $\leq n$, the invariants taking values in any abelian group. The ralation of n-equivalence on knots in S^3 has been characterized by Habiro [1], in terms of C_{n+1}-moves on knots, and Stanford [5], in terms of the pure braid $(n+1)$-commutators. A knot is called n-trivial, if it is n-equivalent to the trivial one. In [2], Kalfagianni and Lin initiated the study of knot invariants of finite type in S^3 via geometric methods. They interpret these invariants as obstructions to a knot's bounding a regular Seifert surface with certain properties. More precisely, in order to describe the classes of n-equivalent knots, they introduced for each $n > 1$ the classes of n-hyperbolic, n-elliptic and n-parabolic knots, and showed that all they are n-trivial. It is unknown however whether the class consisting of all n-hyperbolic, n-elliptic and n-parabolic knots coincides with the one of n-trivial knots. They asked if furthermore, $(n-1)$- and $(n-2)$-hyperbolic knots are n-trivial. We show that in the first case the answer to the question is affirmative, while in the second case this is negative. More precisely, we show that all $(n-1)$-hyperbolic, $n > 2$, are n-trivial and that for each odd integer $n > 3$ there is an $(n-2)$-hyperbolic knot which is not n-trivial. Note that any n-hyperbolic and n-elliptic knots have the trivial Alexander polynomial. We introduce the notion of n-band equivalent knots and study the relationship between the class of all n-hyperbolic and n-elliptic knots and the class of knots, n-band equivalent to the trivial one. We discuss also the problem of how can change the group of a knot, after an application of a simple C_n-move to it, or equivalently, after an insertion of a pure braid n-commutator p_σ [3] in the knot. We provide a sufficient condition under which a simple C_n-move on a knot does not affect the Alexander polynomial of it [4]. Several problems concerning the geometric properties of invariants of finite order are also raised.

References

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