# Invariant Kähler polarizations and geometry of vector fields

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Let $G/K$ be a Riemannian symmetric space. Let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{m}$ be the corresponding invariant splitting of the Lie algebra $\mathfrak{g}$ of $G$. Geometric constructions which come from geometric quantization naturally lead to complex structures defined on the punctured cotangent bundle $T_0^*(G/K) = T^*(G/K) - \{\text{zero section}\}$. Such structure $J_S$ for the spheres was found by Souriau. Later it was observed by Rawnsley, that the length function is strictly plurisubharmonic with respect to the above complex structure $J_S$ and defines the Kähler metric on $T_0^*S^n$ with the canonical symplectic form $\Omega$ as the Kähler form. He also observed that $J_S$ is invariant with respect to the Hamiltonian flow of the length function (the normalized geodesic flow) and used the Kähler polarization $J_S$ to quantize the geodesic flow on the spheres.

Our purpose is to describe $G$-invariant Kähler structures on $T^*(G/K)$. Denote by $\langle , \rangle$ the scalar product on $\mathfrak{m}$ defining the Riemannian metric on $G/K$. Let $S^+(\mathfrak{m}^{\mathbb{C}})$ be the set of all invertible symmetric (with respect to $\langle , \rangle$) operators $s : \mathfrak{m}^{\mathbb{C}} \to \mathfrak{m}^{\mathbb{C}}$ such that $\operatorname{Re} s > 0$ (positive definite).

**Theorem.** *There is one-to-one correspondence between the set of all $G$-invariant Kähler structures $\{(J, \Omega)\}$ on $T^*(G/K)$ and the set $\{P\}$ of all smooth mappings $P : \mathfrak{m} \to S^+(\mathfrak{m}^{\mathbb{C}})$, $w \mapsto P_w$ such that* 1) $(\operatorname{Ad}_k P \operatorname{Ad}_{k^{-1}})|\mathfrak{m} = P_{\operatorname{Ad}_k w}$, $\forall w \in \mathfrak{m}, k \in K$; 2) $[P_w(\xi), P_w(\eta)](w) = -[w, [\xi, \eta]]$ *on* $\mathfrak{m}$, $\forall \xi, \eta \in \mathfrak{m}$, *where we consider the maps* $w \mapsto P_w(\xi)$, $w \mapsto P_w(\eta)$ *as vector fields on* $\mathfrak{m}$.

It is described all mappings $P$ satisfying conditions of Theorem in the following cases: (1) if $\operatorname{rank} G/K = 1$, $\dim G/K \geq 3$; (2) if the corresponding Kähler structure $(J, \Omega)$ is invariant under the normalized geodesic flow. For Hermitian symmetric spaces $G/K$ of rank $\geq 2$ a family of Kähler structures (of mappings $P$) is constructed. As an application of these results we obtain 1) a description of the curvature tensor of the symmetric space $F_4/Spin(9)$ in terms of its $Spin(9)$-structure; 2) new results concerning the hyper-Kähler metric on $T^*(G/K)$ if $G/K$ is a Hermitian symmetric space of rank $\geq 1$ (in this case the natural holomorphic-symplectic structure underlies the hyper-Kähler metric, whose restriction to $G/K$ is the given homogeneous metric).