Notes on extended recurrent and extended quasi-recurrent manifolds

Stanisław Ewert-Krzemieniewski (Szczecin)

Recently M. Prvanović ([P]) introduced a type of manifold (M, g) whose $(0,4)$ curvarture tensor R satisfies

$$\begin{aligned}
&\nabla_Z R(X,Y,U,V) = \\
&A(Z)\left[R(X,Y,U,V) + (\beta-\psi)\,G(X,Y,U,V)\right] + \\
&\tfrac{\beta}{2}\left[A(X)G(Z,Y,U,V) + A(Y)G(X,Z,U,V) + \right. \\
&\left. A(U)G(X,Y,Z,V) + A(V)G(X,Y,U,Z)\right],
\end{aligned} \tag{1}$$

where β, ψ are functions on M, A is a closed form satisfying $\beta A(\frac{\partial}{\partial x^r}) = \frac{\partial \psi}{\partial x^r}$ and $G(X,Y,U,V) = g(Y,U)g(X,V) - g(Y,V)g(X,U)$. She proved that in a neighbourhood of a generic point the associated 1-form A is concircular, i. e.

$$(\nabla A)(X,Y) = Fg(X,Y) + HA(X)A(Y)$$

holds for some function F, H, and found the local form of the metric ([P]). The condition (1) can be considered as a generalisation of the well known notion of a recurrent manifold ($\nabla_Z R = a(Z)R$, ([W])) as well as of generalized recurrent manifold introduced by Dubey ([D])

$$\nabla_Z R(X,Y,U,V) = K(Z)R(X,Y,U,V) + L(Z)G(X,Y,U,V). \tag{2}$$

On the other hand Chaki ([Ch]) introduced and studied a type of manifolds satisfying

$$\begin{aligned}
&\nabla_Z R(X,Y,U,V) = \\
&2a(Z)R(X,Y,U,V) + a(X)R(Z,Y,U,V) + \\
&a(Y)R(X,Z,U,V) + a(U)R(X,Y,Z,V) + a(V)R(X,Y,U,Z)
\end{aligned}$$

known as a pseudo-symmetric (in the sense of Chaki) or quasi-recurrent.

We discuss the condition including the above ones.This can be reduce to the form

$$\begin{aligned}&\nabla_Z R(X,Y,U,V) = \\ &2a(Z)R(X,Y,U,V) + a(X)R(Z,Y,U,V)+ \\ &a(Y)R(X,Z,U,V) + a(U)R(X,Y,Z,V) + a(V)R(X,Y,U,Z) \\ &2b(Z)G(X,Y,U,V) + b(X)G(Z,Y,U,V)+ \\ &b(Y)G(X,Z,U,V) + b(U)G(X,Y,Z,V) + b(V)G(X,Y,U,Z).\end{aligned} \tag{3}$$

We derive the formulas for the Ricci tensor, its covariant derivative and the Riemann curvature tensor. It appears that the manifold is of almost constant curvature. Making use of tensor charcterisation of subprojective spaces ([K]) we prove

Theorem 1 *Let M be a conformally flat manifold, $dimM > 4$, whose curvature satisfy (3). If the curvature tensor R and the 1-form a do not vanish on any dense subset of M, then M is subprojective manifold.*

This enables finding the local form of the metric of manifolds under consideration.

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