INFINITE DIMENSIONAL LIE THEORY BY MEANS OF THE EVOLUTION MAPPING

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Abstract

An infinite dimensional Lie theory is said to be abstract if in the definition of the smooth structure on Lie groups charts are not required. Several abstract settings exist (Souriau, Chen, Omori), but usually they do not correspond to each other. The necessity of them is motivated by important examples and applications.

An infinite dimensional Lie group G with its Lie algebra $\mathfrak{g}$ is called regular if there is a bijective evolution mapping

$$\mathrm{evol}_G^r : C^\infty(\mathbb{R}, \mathfrak{g}) \to \mathfrak{C}^\infty((\mathbb{R}, \mathfrak{o}), (\mathfrak{G}, \mathfrak{e}))$$

such that its evaluation at $1 \in \mathbb{R}$ is smooth. This notion has been introduced by Milnor. A concept generalizing regular Lie groups is proposed. In this concept the smooth structure is defined by means of the evolution mapping evol_G^r. Basic properties of Lie groups can be derived from our definition. New interpretations of the inheritance property, the third Lie theorem, and other facts are possible.

KEYWORDS: infinite dimensional Lie group, regularity

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