SUBMODULES OF VECTOR FIELDS AND ALGEBROIDS

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Abstract

Algebroids and generalized algebroids defined in [3], particularly Courant algebroids considered in [1], define involutive and finite generated submodules of vector fields. But the most known algebroids are the Lie algebroids. They are considered by J. Pradines in [4] in connection with Lie groupoids, giving a coherent generalization of Lie theory. The third theorem of Lie related to the integration of Lie algebroids to Lie groupoids failed from the global viewpoint (Almeida-Molino, 1985, see [2, Theorem 4.4]). Most of people focused in particular to the global integration problem of Lie algebroids to Lie groupoids. Even the integration is not always possible, some particular integrable cases or obstructions regarding the integration were studied.

The algebroids considered in this paper are vector bundles which the module of sections fulfills the conditions of a Lie algebroid, except the Jacobi condition. But as our knowledge is, the relation between algebroids and vector fields, regarding the situation when a submodule of vector fields can be defined by the image of an algebroid, has not been yet studied. We prove that *the necessary and sufficient condition for a submodule of vector fields to be defined by the image of an algebroid is that the module be involutive and finite generated.* It means that the algebroid is a sufficient notion for these modules. As a first case when this situation occurs, we prove that *a singular Riemannian foliation is always defined by an algebroid.* As a second application, the canonical central anchored bundle of regular and singular Riemannian foliations is defined and the cojecture of P. Molino which asserts that *the closures of leaves of a singular Riemannian foliation is also a singular Riemannian foliation* is proved. In fact, Molino has left to prove only that the closures of leaves of a singular Riemannian foliation is a Stefan-Sussmann foliation [2, Chapter 6]; here we fill up this gap.

References

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