MODULAR CLASSES OF ANCHORED MODULES

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The categories of modules with differentials and the vector bundles with differentials, are defined by the first author in [5]. As explained in [4] or [5], they are two categories of vector bundles and two categories of modules; they are also two functors from the each category to the other. Some corresponding functors are induced from the categories of vector bundles with differentials to the corresponding categories of modules with differentials.

As it is proved in [7], there are two functors (called here the *derived functors*) from the two categories of anchored modules which allow linear connections to the two categories of Lie pseudoalgebras; in the presence of linear connections, these functors can be defined on all the categories of modules with differentials. A Lie pseudoalgebra (the derived Lie pseudoalgebra) can be associated with an anchored module which allows a linear connection. The derived Lie pseudoalgebra does not depend on the linear connection, but on the anchor, being an invariant object associated with an anchored module which allows a linear connection. Considering also the correspondences of morphisms, one define two pseudofunctors (i.e. a functor except sending the identity in the identity) and two natural functors (called the derived functors) respectively on the two categories of anchored modules which allow linear connections to the corresponding two categories of Lie pseudoalgebras.

We prove in the paper that the construction of the derived functors can be also performed using an other way, which is more suitable for vector bundles [6].

We show that a linear connection lifts on the derived module to a curvature free connection. The Picard groups related to anchored modules as well as the modular classes of preinfinitesimal modules and almost Lie structures are defined. The modular classes defined here agrees in a certain sense with the modular class of a Lie pseudoalgebra defined by J. Huebschmann [3].

References

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