METRIC-POLYNOMIAL STRUCTURES FROM GRAVITATIONAL LAGRANGIANS

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Abstract

We study these metric-polynomial structures on manifold which arises as extremals of the Palatini variational principle for some class of gravitational Lagrangians. They can be described in the following way.

Let (M, g, Γ) be a n-dimensional (pseudo-) Riemannian manifold (M, g) equipped with a symmetric (i.e. torsion-free) connection Γ. Define a $(1,1)$ tensor field concomitant

$$S^\mu_\nu \equiv S^\mu_\nu(g, \Gamma) \doteq g^{\mu\alpha} R_{(\alpha\nu)}(\Gamma)$$

where $R_{(\alpha\nu)}(\Gamma)$ denotes the symmetric part of the Ricci tensor of Γ. Consider a family of scalar-valued concomitant

$$s_k \equiv s_k(g, \Gamma) \doteq \operatorname{tr} S^k$$

the so-called Ricci scalar of order k, $k = 1, \cdots, n$. For F being an arbitrary (differentiable) real-valued function of n-variables one can define the corresponding Lagrangian of the Ricci type

$$L_F \doteq \sqrt{g}\, F(s_1, \cdots, s_n)$$

Applying now the Palatini variational principle we arrive to the following results:

- the connection Γ is a Levi-Civita connection for some pseudo-Riemannian Einstein metric h,
- the $(1,1)$ tensor field S satisfies a polynomial equation

  $$w_F\,(S) = 0$$

  for some polynomial function $w_F(t)$ of constant coefficients,
- the metric h and polynomial structure S are compatible in a sense

  $$h(SX, Y) = h(X, SY)$$

  for each pair of tangent vector fields (X, Y) on M.

In particular, for f being a function of one variable the Lagrangian $L_f = \sqrt{g}\, f(s_1)$ reconstructs the Einstein theory. For $L_f = \sqrt{g}\, f(s_2)$, besides the Einstein equation, one gets a pseudo-Riemannian almost product structure ($S^2 = I$) and/or an almost-complex anti-Hermitian structure ($S^2 = -I$). Some other examples will be also considered.

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