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The L^2-invariants and non-singular Morse-Smale flows on manifolds

A smooth flow φ_t on smooth closed manifold M^n is called non-singular Morse-Smale if: the chain-recurrent set of φ_t consist of finite number hyperbolic closed orbit; the unstable manifold of any closed orbit has transversal intersection with the stable manifold of any closed orbit.

Definition 0.1 *The i-th Morse S^1-number of manifold M^n is the minimum number of closed orbits of index i taken over all non-singular Morse-Smale vector fields with untwisted closed orbits on manifold M^n. Denote the i-th Morse S^1-number of manifold M^n by $\mathcal{M}_i^{S^1}(M^n)$.*

Let $N^{n-1} \subset M^n$ be a closed submanifold of the closed manifold M^n. We shall say that (M^n, N^{n-1}) form **admissible par of the level** i if : $M^n \backslash N^{n-1}$ consist from two components, closing which we denote by M_{un} and M_{up} ; M_{un} (M_{up}) have homotopy type of k-dimensional (l-dimensional) CW-complex, where $k \leq i, l \leq n-i-1$.

Definition 0.2 *The admissible par (M^n, N^{n-1}) of level i we call χ-minimal if the number $(-1)^i\chi(M_{un})$ is minimal between all admissible pars of the level i of the manifold M^n. The value of number $(-1)^i\chi(M_{un})$ for minimal admissible par of the level i of the manifold M^n we shall call i-th Euler characteristic of manifold M^n and shall denote by $\chi_i(M^n)$.*

Lemma 0.1 *Let M^n be a closed smooth manifold ($n \geq 9$).On M^n there is ordered Morse function $f : M^n \longrightarrow [0, n]$ such that: f have one critical point of index 0 (1) ; all critical points of index λ of the function f lie on $f^{-1}(\lambda)$; f have minimal number critical points of fixed index i.If $4 \leq i \leq n-5$, then the par $(M^n, f^{-1}(i + 1/2))$ is χ-minimal admissible par of the level i .*

Lemma 0.2 *Let (M^n, N^{n-1}) χ-minimal admissible par of the level i. If $n \geq 9$ and $4 \leq i \leq n-5$, then $\chi_i(M^n)$ is invariant of homotopy type of M^n and equal $\chi_i(M^n) = (-1)^i \sum_{j=0}^{i} dim_{N(G)}(H^i_{(2)}(M^n)) + \widehat{S}^{i+1}_{(2)}(M^n)$.*

Теорема 0.1 *Let M^n, $n \geq 9$, be arbitrary closed manifold with zero Euler characteristic, then i-th Morse S^1-number of manifold M^n for $4 \leq i \leq n-5$ is the invariant of homotopy type of M^n and equal $\mathcal{M}_i^{S^1}(M^n) = \rho(\chi_i(M^n))$, where $\rho : \mathbb{Z} \longrightarrow \mathbb{N}$ such that $\rho(x) = 1/2(x+ \mid x \mid)$.*