On universality of Kaehlerian manifold

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Let (M, g) be a smooth n-dimensional Riemannian manifold. We have considered the almost Hermitian structure $(J, \widehat{g})$ on TM constructed with help of a Riemannian metric g and the second fundamental tensor field h of $(J, \widehat{g})$, see **[1]**. So, the following injective mapping has been obtained

$$E : R(M) \to AH(M) : g \mapsto (J, \widehat{g}),$$

where $R(M)$ is the set of all Riemannian metrics on M and $AH(M)$ is the set of all almost Hermitian structures on TM.

For any point $p \in M$ we can consider such a set $\{\delta(p)\}$ of positive numbers that the mapping $exp_{|U(\delta(p))}$ is defined and injective on $U(\delta(p)) \subset T_pM$. Let $\varepsilon(p) = sup\{\delta(p)\}$.

Lemma, [2]. *The mapping* $M \to R_+ : p \mapsto \varepsilon(p)$ *is continuous on* M.

Proposition. *There exists a normal tubular neighborhood* $Tb(M, \varepsilon(p)) = \bigcup_{p \in M} D(p; \varepsilon(p))$ *in* TM *with respect to* $\widehat{g} = E(g)$ *on* TM *induced by some* $g \in R(M)$, *where* M *is considered as the null section* O_M *in* $TM (M \ni p \leftrightarrow O_p \in O_M \subset TM)$ *and* $\varepsilon(p)$ *is continuous function on* M.

All the constructions announced in **[3]** and considered in **[4]** for the case $\varepsilon = const$ can be generalized for the case of positive continuous function $\varepsilon(p)$ on M.

Theorem. *Let* M *be a smooth real manifold and* $Tb(M, \varepsilon(p))$ *be the corresponding normal tubular neighborhood in* TM *with respect to some* $\widehat{g} = E(g)$ *on* TM.

Then $M(O_M)$ *is a totally geodesic submanifold of the Kaehlerian manifold* $(Tb(M, \frac{\varepsilon(p)}{2}, \overline{J}, \overline{g}))$, *where the almost Hermitian structure* $(\overline{J}, \overline{g})$ *is the deformation of the structure* $(J, \widehat{g})$, *see* ***[3],[4]***.

So, any smooth real manifold of dimension n *can be embedded in a Kaehlerian manifold of dimension* $2n$ *as a totally geodesic submanifold.*

References

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