ABOUT TOPOLOGICAL EQUIVALENCE OF SOME FUNCTIONS

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Abstract. The number of topologically non-equivalent special functions f on the circle is calculated. Necessary and sufficient condition for topological equivalence of two pseudoharmonic functions $F, G : D^2 \to R$ is formulated in the terms of their combinatoric diagrams.

Keywords. pseudoharmonic functions, a combinatoric invariant, a topological equivalence.

1) Let $f : S^1 \to R$ be a continuous function with a finite number of local extrema.

Definition 1 *A_n updown sequence of length n is a sequence of integers $i_1 < i_2 > ...i_n$, such that $\{i_1, ..., i_n\} = \{1, ..., n\}$.*

If $n = 2r - 1$ then the number of such sequences is equal to the tangent number T_r (see [1]).

Definition 2 *A function f is called special if arbitrary two local extrema of f correspond to two different values of f.*

Theorem 1 *The number of topologically non-equivalent special functions f on the circle with $2n$ local extrema is equal to T_n.*

Denote by $G(n)$ the number of topologically non-equivalent special functions f on the circle with $(2n + 2)$ local extrema. In [4] proven that the following estimate is true:

$$\lim_{n\to\infty} \frac{\log G(n)}{n \log n} = 2.$$

2) Let $F : D^2 \to R$ be a pseudoharmonic function, where D^2 is a domain bounded by a Jordan curve. Recall that function F is pseudoharmonic if there is a homeomorphism φ of domain D^2 onto itself such that $F \circ \varphi$ is harmonic. We will consider a pseudoharmonic function which satisfies the following boundary condition: $F|_{S^1}$ is continuous with a finite number of local extrema. It is known that such a function F has only saddle critical points in the interior of D^2 (see [2], [5]).

Definition 3 *The value c of F is called regular (critical) if the connected components of level curves $F^{-1}(c)$ contain critical points (don't contain critical points and are isomorphic to a disjoint union of segments).*

Definition 4 *The value c of F is semiregular if it is neither regular nor critical.*

Let us describe the combinatoric diagram of function F. At first we construct the (Kronrod-Reeb graph) Γ_{K-R} of $F|_{\partial D^2}$. Then, we add to the graph $\Gamma_{K-R}(F|_{\partial D^2})$ level curves of sets $\widehat{F}^{-1}(a_i)$ and $\widehat{F}^{-1}(c_i)$, where $\widehat{F}^{-1}(a_i)$, $\widehat{F}^{-1}(c_i)$ are subsets of $F^{-1}(a_i)$, $F^{-1}(c_i)$ which contain only critical and semiregular points. Then

$$P(F) = \Gamma_{K-R}(F|_{S^1}) \bigcup_i \widehat{F}^{-1}(a_i) \bigcup_j \widehat{F}^{-1}(c_j),$$

where every a_i is critical value and every c_j is semiregular value. By using the values of function, we can put the strict partial order on vertices in the following way: $v_1 > v_2 \iff F(x_1) > F(x_2)$, where $v_1, v_2 \in P(F)$, x_1, x_2 are points which correspond to vertices v_1, v_2. If $F(x_1) = F(x_2)$ then v_1 and v_2 are non-comparable (see [3]).

Theorem 2 *Two pseudoharmonic functions F and G are topologically equivalent if and only if there is an isomorphism $\varphi : P(F) \to P(G)$ between their combinatoric diagrams which preserves the strict partial order given on them.*

References

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