Modified Hochschild and Periodic Cyclic Homology.

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1 Abstract

It is known that the Hochschild and (periodic) cyclic homology of Banach algebras are either trivial or not interesting. To correct this deficiency, Connes had produced the *entire cyclic cohomology*. The entire cyclic cochains are elements of the infinite product (b, B) cohomology bi-complex which satisfy a certain bidegree asymptotic growth condition. The entire cyclic cohomology is a natural target for the asymptotic Chern character of $\theta-$summable Fredholm modules. More recently, Puschnigg introduced the *local cyclic cohomology* based on precompact subsets of the algebra in an inductive limits system setting.

The main purpose of this paper is to create an analogue of the Hochschild and periodic cyclic homology which gives the right result (i.e. the ordinary Z_2-graded Alexander-Spanier co-homology of the manifold) when applied, at least, onto the algebra of continuous functions on topological manifolds and $CW-$complexes. This is realized by replacing the Connes periodic bi-complex (b, B), by the bi-complex $(\tilde{b}, d)$, where the operator $\tilde{b}$ is obtained by blending the Hochschild boundary b with the Alexander-Spanier boundary d; the operator $\tilde{b}$ anti-commutes with the operator d. The homologies of these complexes will be called *modified Hochschild*, resp. *modified periodic cyclic homology*.

Our construction uses in addition to the algebraic structure solely the *locality* relationship extracted from the topological structure of the algebra.

The modified periodic cyclic homology is invariant under *continuous* homotopies, while the others are invariant at *smooth* homotopies (diffeotopies) only.

The modified Hochschild and periodic cyclic homology are directly connected to the Alexander-Spanier cohomology.