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THE BASES OF DIFFERENTIAL GEOMETRY OF VECTOR FIELD IN n-DIMENSIONAL SPACE OF AFFINE CONNECTION.

Abstract

Series of invariant straights, hypersurfices and hyperquadrics for vector field of n-dimensional space of affine connection have been built. The "splitting" of geometric models of vector field in transition from affine space to space of affine connection.

Classic space of affine connection $\boldsymbol{A_{n,n}}$ is determined by system $(n+1)^2$ forms ω^2 and ω_j^i, which ordered to equations of structure

$$\begin{aligned} D\omega^i &= \left[\omega^k \omega_k^i\right] + R_{jk}^i \left[\omega^i \omega^k\right], \\ D\omega_i^j &= \left[\omega_i^k \omega_k^j\right] + R_{ikl}^j \left[\omega^k \omega^l\right] \end{aligned} \qquad (1)$$

In equations (1) values R_{jk}^i are sidelong symmetric to upper indexes and in a set they form tenzor of steepness of $\boldsymbol{A_{n,n}}$ space values R_{jk}^i are sidelong symmetric to indexes *k*, *l* and they form tenzor of steepness $\boldsymbol{A_{n,n}}$ space.

Definition. Vector field in space $\boldsymbol{A_{n,n}}$ is called the correspondence in which every point ***A(u)*** base of space $\boldsymbol{A_{n,n}}$ corresponds definite vector $\vec{v}(u)$ which belongs to n-dimensional affine space $\boldsymbol{A_n(u)}$ related to moving reper $T_n = \left\{\vec{A}(n), e_\alpha(n)\right\}$. This space as it is obvious is a layer over point ***A(u)***.

System of differential equations of vector field in reper of zero order (starting point ***A(u)*** of vector field coincides with the end of vector $\vec{A}$, and vector $\vec{v}$ coincides with $\vec{v}_n$ en) have the form

$$\omega_n^\alpha = a_{n\beta}^\alpha \omega^\beta \quad (\alpha, \beta, \gamma ... \overline{1,n}) \qquad (2)$$

Continuing the system of differential equations (2) we'll receive the system of differential equations of fundamental object of the first order of vector field of space $\boldsymbol{A_{n,n}}$ in the form

$$d\Lambda^{\alpha}_{n\beta} = \Lambda^{\alpha}_{n\gamma}\omega^{\gamma}_{\beta} - \Lambda^{\gamma}_{n\beta}\omega^{\alpha}_{\gamma} + \Lambda^{\alpha}_{n\beta\gamma}\omega^{\gamma} \tag{3}$$

where

$$\Lambda^{\alpha}_{n[\beta\gamma]} + \Lambda^{\alpha}_{n\delta}R^{\delta}_{\gamma\beta} + R^{\alpha}_{n\beta\gamma} = 0\,, \tag{4}$$

Continuing the system of differential equations (3) we'll have

$$d\Lambda^{\alpha}_{n\beta\gamma} = \Lambda^{\alpha}_{n\delta\gamma}\omega^{\delta}_{\beta} + \Lambda^{\gamma}_{n\beta\delta}\omega^{\alpha}_{\gamma} - \Lambda^{\delta}_{n\beta\gamma}\omega^{\alpha}_{\delta} + \Lambda^{\alpha}_{n\beta\gamma\delta}\omega^{\delta}\,. \tag{5}$$

Succession of fundamental objects $\{\Lambda^{\alpha}_{n\beta,}, \Lambda^{\alpha}_{n\beta\gamma}, \Lambda^{\alpha}_{n\beta\gamma\delta}, \ldots\}$ lies in the basis of differential geometry of vector field in space $\boldsymbol{A_{n,n}}$.

Remark: apart from n-dimensional affine space tenzors $\Lambda^{\alpha}_{n\beta\gamma}, \Lambda^{\alpha}_{n\beta\gamma\delta}, \ldots$ loose symmetric properties on two down last indexes.

Let's consider values Λ^{α}_{nn}. If their differential equation has form

$$d\Lambda^{\alpha}_{nn} = -\Lambda^{\beta}_{nn}\omega^{\alpha}_{\beta} + \Lambda^{\alpha}_{nn}\omega^{\beta} \tag{6}$$

the straight put by the equation

$$x^{\alpha} = t\Lambda^{2}_{nn} \tag{7}$$

relatively to local reper is invariant.

We'll build values $V^{\beta}_{n\alpha}\Lambda^{\gamma}_{n\beta} = \delta^{\gamma}_{\alpha}$, in condition, that $def\ //\ \Lambda^{\alpha}_{n\beta}\ //\ \neq 0$.

With the help of value $V^{\alpha}_{n\beta}$ consider values

$$V^{i} = -V^{\alpha}_{n\beta}\Lambda^{\beta}_{nn}\,. \tag{8}$$

Their differential equation has form

$$dV^{\alpha} = -V^{\beta}_{nn}\omega^{\alpha}_{\beta} + V^{\alpha}_{\gamma}\omega^{\gamma} \tag{9}$$

Thus, the straight introduced by the equation

$$x^{\alpha} = tV^{\alpha} \tag{10}$$

is also invariant.

With the help of fundamental objects of the first and second order $\Lambda^{\alpha}_{n\beta}, \Lambda^{\alpha}_{n\beta\gamma}$ built values in succession

$$\Lambda^{\beta}_{n\beta\alpha} = \overset{1}{\Lambda}_{n\alpha} \quad d\overset{1}{\Lambda}_{n\alpha} = \overset{1}{\Lambda}_{n\beta}\,\omega^{\beta}_{\alpha} + \overset{1}{\Lambda}_{n\alpha\beta}\,\omega^{\beta} \tag{11}$$

$$\Lambda^{\beta}_{n\alpha\beta} = \overset{2}{\Lambda}_{n\alpha} \quad d\overset{2}{\Lambda}_{n\alpha} = \overset{2}{\Lambda}_{n\beta}\,\omega^{\beta}_{\alpha} + \overset{2}{\Lambda}_{n\alpha\beta}\,\omega^{\beta} \tag{12}$$

These values define invariant hypersurfice which don’t cross the point A:

$$\overset{1}{\Lambda}_{n\alpha} x^{\alpha} + 1 = 0 \qquad (13)$$

$$\overset{2}{\Lambda}_{n\alpha} x^{\alpha} + 1 = 0 \qquad (14)$$

In case of space $\boldsymbol{A_n}$ these hypersurfices coincide. Thus, we deal with “splitting” of invariant hypersurfice which has been mentioned by D.M. Sintsov.

We’ll built the following formula

$$\begin{aligned} &\Lambda_n = \Lambda^{\alpha}_{n\alpha}, \quad \delta\Lambda_n = 0 \\ &\overset{1}{\Lambda}_{n\alpha\beta} = \Lambda^{\gamma}_{n\beta\alpha} + \overset{1}{\Lambda}_{n\gamma} \\ &\overset{2}{\Lambda}_{n\alpha\beta} = \Lambda^{\gamma}_{n\beta\alpha} + \overset{2}{\Lambda}_{n\gamma} \end{aligned} \qquad (15)$$

With their help we’ll build hyperquadrics

$$\begin{aligned} &\overset{1}{\Lambda}_{n\alpha\beta} x^{\alpha} x^{\beta} + 2\overset{1}{\Lambda}_{n\alpha} x^{\alpha} + \Lambda = 0 \\ &\overset{2}{\Lambda}_{n\alpha\beta} x^{\alpha} x^{\beta} + 2\overset{2}{\Lambda}_{n\alpha} x^{\alpha} + \Lambda = 0 \end{aligned} \qquad (16)$$

In case of n-dimensional affine space these hyperquadrics coincide.