A note on the reconstruction problem for factorizable homeomorphism groups and foliated manifolds

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Let G be a group of homeomorphisms of a topological space X, and H be a group of homeomorphisms of a topological space Y. I shall explain some general theorem which states that under suitable assumptions on X, Y, G and H, every isomorphism φ between G and H is induced by some homeomorphism τ between X and Y. That is,

$$\varphi(g) = \tau \circ g \circ \tau^{-1}, \qquad \text{for every } g \in G.$$

The general method is used in order to prove the theorems described below. For a group G of homeomorphisms of a regular topological space X and an open $U \subseteq X$, set $G\lfloor U \rfloor := \{g \in G \mid g \restriction (X \setminus U) = \mathrm{Id}\}$. We say that G is a *factorizable group of homeomorphisms*, if for every open cover $\mathcal{U}$ of X, $\bigcup_{U \in \mathcal{U}} G\lfloor U \rfloor$ generates G.

Theorem A Let G, H be factorizable groups of homeomorphisms of X and Y respectively, and suppose that G, H do not have fixed points. Let φ be an isomorphism between G and H. Then there is a homeomorphism τ between X and Y such that $\varphi(g) = \tau \circ g \circ \tau^{-1}$ for every $g \in G$.

Theorem A strengthens known theorems in which the existence of τ is concluded from the assumption of factorizability and some additional assumptions.

Theorem B For $\ell = 1, 2$ let (X_ℓ, Φ_ℓ) be countably paracompact foliated (not necessarily smooth) manifolds and G_ℓ be either the group of leaf-preserving homeomorphisms of (X_ℓ, Φ_ℓ) or the group of homeomorphisms of (X_ℓ, Φ_ℓ) which take every leaf to itself. Let φ be an isomorphism between G_1 and G_2. Then there is a leaf-preserving homeomorphism τ between X_1 and X_2 such that $\varphi(g) = \tau \circ g \circ \tau^{-1}$ for every $g \in G_1$.