Around Birkhoff Theorem

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Let X be a topological space, $f : X \to X$. A point $x \in X$ is said to be *recurrent* if, for every neighbourhood U of x and every $n \in \omega$, there exists $m > n$ such that $f^m(x) \in U$. By Birkhoff Theorem, every continuous mapping $f : X \to X$ of a compact space has a recurrent point.

We say that a topological space X is *totally recurrent* if **every** mapping $f : X \to X$ has a recurrent point.

Theorem. *A Hausdorff space X is totally recurrent if and only if X is either finite or a one-point compactification of an infinite discrete space.*

Problem 1. Detect all totally recurrent T_1-spaces.

Problem 2. Characterize all Hausdorff spaces in which every injective (resp. surjective, bijective) self-mapping has a recurrent point.

Problem 3. Describe all Hausdorff spaces in which every continuous self-mapping has a recurrent point.

S.Kolyada suggested the following question.

Problem 4. Which compact metric spaces X admit a structure of minimal dynamical system, i.e. a continuous mapping $f : X \to X$ such that the orbit $\{f^n(x) : n \in \omega\}$ of each point $x \in X$ is dense in X.L