NODE POINTS OF SEN-WITTEN EQUATION
AND POSITIVE ENERGY PROBLEM IN GENERAL RELATIVITY

V. Pelykh
pelykh@lms.lviv.ua, IAPMM NASU
3B Naukova Str., Lviv, 79601, Ukraine

The existence of the tensor method for the positive energy theorem (PET) proof in general relativity was unanimously declared impossible owing to existence of node points for Dirac equation in $\mathbb{R}^3$ or Sen-Witten equation on asymptotically flat manifolds. We give the correct tensor proof of the PET on the base developed by us a new approach for establishing the conditions of the Dirichlet problem solvability and zeros absence for general-covariant and locally $SU(2)$-covariant elliptic system of equations, which contains in particular case Dirac and Sen-Witten equation.

We obtain the new condition of the Dirichlet problem solvability and the condition of zeros absence for solutions of this general-covariant and locally $SU(2)$-covariant system

$$\frac{1}{\sqrt{-h}}\frac{\partial}{\partial x^{\alpha}}\left(\sqrt{-h}h^{\alpha\beta}\frac{\partial}{\partial x^{\beta}}u_A\right) + C_A{}^B u_B = 0, \tag{1}$$

where $h^{\alpha\beta}$ — components of the metric tensor in V^3; they are arbitrary real functions of independent real variables x^{α}, continuous in Ω and the quadratic form $h^{\alpha\beta}\xi_{\alpha}\xi_{\beta}$ is negatively defined. The unknown functions u_A of independent variables x^{α} are the elements of complex vector space $\mathbf{C}^2$, in which the skew symmetric tensor ε^{AB} is defined, and the group $SU(2)$ acts. $C_A{}^B$ is Hermitian $(1,1)$ spinorial tensor.

On this basis we prove further that Sen-Witten equation have not node points if initial data set is asymptotically flat, dominant energy condition is fulfilled and at least one component of Sen-Witten spinor field asymptotically nowhere equals to zero.

Our work is next substantial argument after [1] in favour of geometrical nature of the Sen-Witten spinor field.

References

[1] Pelykh V. Comment on "Self-dual teleparallel formulation of general relativity and the positive energy theorem" // Phys.Rev.D. – 2005. – Vol. 72. – 108502.