Surgery on stratified manifolds

Yuri V. Muranov

ymuranov@mail.ru

Abstract

The question of whether an element x of the Wall group $L_n(\pi)$ is the surgery obstruction of a degree-one normal map of closed manifolds with $\pi_1(X) = \pi$ is one of the basic problems in surgery theory. This question is equivalent to studying if x belongs to the image of the assembly map $A : H_n(B\pi, \mathbf{L}_\bullet) \to L_n(\pi)$. Note that any element $x \in L_n(\pi)$ is represented by a normal map of closed manifolds with boundaries.

Let X be a stratified manifold in the sense of Browder and Quinn. A pair $Y^{n-q} \subset X^n$ of closed manifolds in the sense of Ranicki is the simplest case of a stratified manifold. In this case, the splitting obstruction groups $LS_{n-q}(F)$ are defined, where

$$F = \begin{pmatrix} \pi_1(\partial U) & \to & \pi_1(X \setminus Y) \\ \downarrow & & \downarrow \\ \pi_1(U) & \to & \pi_1(X) \end{pmatrix}$$

is the push-out square of fundamental groups for the splitting problem. All elements of the groups $LS_*(F)$ are realized by maps of manifolds with boundaries. The problem of realizability splitting obstructions by homotopy equivalences of closed manifolds closely relates to computing of the Assembly map.

We give a generalization of the splitting theory to the case of stratified manifolds. The structure of a stratified manifold provides new algebraic approach to computing of the Assembly map. In the particular case of a Browder-Livesay filtration this approach closely relates to the approach that is based on conception of iterated Browder-Livesay invariants developed by Hambleton and Kharshiladze. We describe new results in surgery on filtered manifolds and relations to classical surgery theory. We consider several examples and give an application of our results to the closed manifolds surgery problem and to the problem of realization of splitting obstructions by simple homotopy equivalences of closed manifolds.

ACKNOWLEDGEMENTS. The author is partially supported by the RFFI grant No. 05–01–00993.