

Hyperspaces of Riemannian manifolds related to the Hausdorff dimension

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Different authors (see, e.g. [1–3]) considered the hyperspaces of compact subsets and subcontinua of given Lebesgue dimension. In particular, in [2] the topology of the system of hyperspaces of given Lebesgue dimension in the Hilbert cube is described. In [3] this result is extended over the case of hyperspaces of countable infinite products of nondegenerated Peano continua and in [1] over the case of a Peano continuum in which every open set contains sets of arbitrary finite dimension.

The author [4, 5] obtained counterparts of the above mentioned results for the hyperspaces of compacta and continua of given Hausdorff dimension in the cube $[0, 1]^n$. The aim of the talk is to extend these results over the case of the hyperspaces of Riemannian manifolds.

By Q we denote the Hilbert cube $[-1, 1]^\omega$, and by $B(Q) = \{(x_i) \in Q \mid x_i \in \{-1, 1\} \text{ for some } i\}$ the pseudoboundary of Q . Further, $HD_{>\gamma}(X)$ ($HD_{>\gamma}^c(X)$) is the hyperspace of compacta (continua) X of the Hausdorff dimension $> \gamma$.

The following is the main result of the talk.

Theorem. *Let $n \in \mathbb{N}$, X be an n -dimensional compact connected Riemannian manifold, and Γ be some countable ordered set.*

(1) *If $\Gamma \subset [0, n)$ then there is a homeomorphism $\alpha: \exp(X) \rightarrow Q^\Gamma$ such that for every $\gamma \in \Gamma$*

$$\alpha[HD_{>\gamma}(X)] = \bigcup_{\gamma' \geq \gamma} \left(\prod_{\gamma'' \neq \gamma'} Q_{\gamma''} \times B(Q)_{\gamma'} \right).$$

(2) *If $n \geq 2$ and $\Gamma \subset [1, n)$ then there is a homeomorphism $\beta: \exp_c(X) \rightarrow Q^\Gamma$ such that for every $\gamma \in \Gamma$*

$$\beta[HD_{>\gamma}^c(X)] = \bigcup_{\gamma' \geq \gamma} \left(\prod_{\gamma'' \neq \gamma'} Q_{\gamma''} \times B(Q)_{\gamma'} \right).$$

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