PRESENTATION FOR THE FUNDAMENTAL GROUPS OF ORBITS OF MORSE FUNCTINOS ON SURFACES

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Let M be a compact surface, P either a real line or a circle, and $f : M \to P$ a Morse mapping. Denote by O_f the right orbit of f, i.e. the orbit of f with respect to the action of the group of diffeomorphisms $Diff(M)$ of M defined by the formula:

$$h \cdot f = f \circ h^{-1}, \qquad h \in Diff(M).$$

(1) We show that O_f is homotopy equivalent to some covering space of the n-th configuration space of M, where n is a total number of critical points of f.

It was proved by the author earlie, that for aspherical surfaces, i.e. all surfaces except 2-sphere and projective plane, the orbit O_f is aspherical. Then (1) implies that $\pi_1 O_f$ is a subgroup of the n-th braid group of M.

(2) We also construct a finite presentation for the group $\pi_1 O_f$ for the case when M is orientable of genus $g \geq 2$.

It turns out that this presentation has similarity with the presentation for Artin groups. In particular, analogues of $K(\pi, 1)$ conjecture and Tits conjecture on squares of generators in Artin group can easily be formulated and proved for $\pi_1 O_f$.

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