Remarks on Extended Operator Calculus

Ewa Krot-Sieniawska

Institute of Computer Science, University in Bialystok ,
PL-15-887 Bialystok, ul. Sosnowa 64, Poland
e-mail: ewakrot@ii.uwb.edu.pl

In mathematics, before the 1970s, the term umbral calculus (or Finite Operator Calculus) was understood to mean the surprising similarities between otherwise unrelated polynomial equations, and certain shadowy techniques that can be used to 'prove' them. These techniques were introduced in the 19th century and are sometimes called Blissard's symbolic method, and sometimes attributed to James Joseph Sylvester, or to Edouard Lucas. In the 1970s, Gian-Carlo Rota, and others [1-5] developed the umbral calculus as the Finite Operator Calculus by means of linear operators on spaces of polynomials. Currently, umbral calculus is understood primarily to mean the study of Sheffer sequences, including polynomial sequences of binomial type and Appell sequences. Already since thirties of XX-th century it had been realized that operator methods might be extend to the use of any polynomial sequences instead of these of binomial type only. The very foundations of such an extensions were laid in 1937 by M.Ward, [6] . The next major contributions we owe to Viskov [7, 8] and Markowsky [9]. The main statements of ψ- extended Rota's finite operator calculus were given by A.K.Kwaśniewski [10-12]. I'll present some topics related to this theory.

References

[1] G.-C.Rota and R.Mullin: *On the foundations of combinatorial theory III. Theory of binomial enumeration.* In B. Harris, editor, Graph theory and its applications, pages 213. Academic Press, 1970.

[2] G.-C. Rota, D. Kahaner, and A. Odlyzko: *On the foundations of combinatorial theory VII. Finite operator calculus.* J. Math. Anal. Appl., 42:684;760, 1973.

[3] G.-C. Rota: Finite operator calculus. Academic Press, New York, 1975.

[4] S.M. Roman and G.-C. Rota :*The umbral calculus.* Adv. Math., 27:95;188, 1978.

[5] S.M. Roman: The Umbral Calculus. Academic Press, 1984.

[6] M. Ward: *A calculus of sequences.* Amer. J. Math.**58**, 255 (1936).

[7] O. V. Viskov: *Operator characterization of generalized Appell polynomials* Soviet Math. Dokl. **16**, 1521 (1975).

[8] O. V. Viskov: *On bases in the space of polynomials*, Soviet Math. Dokl.**19**, 250 (1978).

[9] G. Markowsky: *Differential operators and the theory of binomial enumeration.* J. Math. Anal. Appl. **63**, 145 (1978).

[10] A. K. Kwaśniewski: *Towards* ψ *-extension of Finite Operator Calculus of Rota* Rep. Math. Phys. **47**, 305 (2001).

[11] A. K. Kwaśniewski:*On Simple Charactrrisation of Sheffer* ψ*-polynomials and Related Proposition of the Calculus of Sequences* Bulletin de la Soc. des Sciences et de Lettres de Lodz ; 52, Ser. Rech. Deform. 36 (2002) pp.45-65 .

[12] A.K.Kwaśniewski: *Main theorems of extended finite operator calculus* Integral Transforms and Special Functions Vol. 14, No 6, pp. 499-516, 2003 Taylor & Francis Online Journal

[13] E.Krot: ψ *-extensions of q-Hermite and q-Laguerre Polynomials - properties and principal statements* Czech. J. Phys. Vol.51(2001) No12, p. 1362-1367.

[14] E.Krot: An Introduction to Finite Fibonomial Calculus, CEJM 2(5)2005 p.754-766, ArXiv: math.CO/0503218

[15] E.Krot: *The first Ascent into the incidence algebra of the Fibonacci cobweb poset* Adv.Stud. in Cont. Math. 11(2005), No. 2, p.179-184