Examples of multiple solutions for the Yamabe problem on scalar curvature

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Abstract: In the conformal class of a Riemannian metric on a compact connected manifold, there exists at least one metric with constant scalar curvature. In the case with positive scalar curvature, there may be many (non-homothetic) metrics with constant scalar curvature in a conformal class. R. Schoen gave a beautiful example of that phenomenon for a one-parameter family of metrics on $S^1 \times S^{n-1}$. This construction may be generalized on products $S^1 \times N$, where N is a compact connected Riemannian manifold with positive constant scalar curvature. A unique (ordinary) differential equation, depending only on the dimension, is the key to that construction. We will give some details on the solutions of that equation and study their behavior in a one-parameter family.