Hyperspaces of max-plus convex sets which are infinite-dimensional manifolds

Lidia Bazylevych, National University “Lviv Polytechnica”

The classical result by Nadler, Quinn and Stavrokas [1] asserts that the hyperspace of convex compact subsets in $\mathbb{R}^n$, $n \geq 2$, is a contractible Q-manifold. Recall that a Q-manifold is a manifold modeled on the Hilbert cube $Q = [0,1]^\omega$.

Let $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$. Given $x, y \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$, we denote by $x \oplus y$ the coordinatewise maximum of x and y and by $\lambda \odot x$ the vector obtained from x by adding λ to every its coordinate. A subset A in $\mathbb{R}^n$ is said to be *tropically convex* if $\alpha \odot a \oplus \beta \odot b \in A$ for all $a, b \in A$ and $\alpha, \beta \in \mathbb{R}_{\max}$ with $\alpha \oplus \beta = 0$. The tropical convexity (or max-plus convexity, in another terminology) was introduced in [2].

The main result states that the hyperspace of compact max-plus convex sets in $\mathbb{R}^n$, $n \geq 2$, is a contractible Q-manifold $Q \setminus \{*\}$. This is a max-plus counterpart of the mentioned result from [1].

We conjecture that, for a nonempty open subset U of $\mathbb{R}^n$, $n \geq 2$, the hyperspace of compact max-plus convex sets contained in U is homeomorphic to the Q-manifold $Q \times [0,1) \times U$. The corresponding result for the hyperspace of compact convex sets is proved by L. Montejano [3].

1. S.B. Nadler, Jr., J. Quinn , N.M. Stavrokas, Hyperspace of compact convex sets, *Pacif. J. Math.* 1979. V. 83. P. 441–462.

2. G. L. Litvinov, V. P. Maslov, and G. B. Shpiz. *Idempotent functional analysis: An algebraic approach.* Translated from Matematicheskie Zametki, vol. 69, no. 5, 2001, pp. 758–797.

3. L. Montejano, *The hyperspace of compact convex subsets of an open subset of* $\mathbb{R}^n$*,* Bull. Pol. Acad. Sci. Math. V.35, No 11-12(1987), 793–799.