HOMEOMORPHISM GROUPS OF NON-COMPACT MANIFOLDS

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For a paracompact space X by $H(X)$ we denote the homeomorphism group of X endowed with the Whitney (or else graph) topology generated by the base consisting of the sets $\Gamma_U = \{h \in H(X) : \Gamma_h \subset U\}$ where U runs over open subsets of X and $\Gamma_h = \{(x, h(x)) : x \in X\}$ stands for the graph of h. Let $H_c(X)$ be the connected component of $H(X)$ containing the identity homeomorphism.

Theorem 1. *For any non-compact connected 2-manifold M the homeomorphism group $H_c(M)$ is homeomorphic to $l_2 \times \mathbb{R}^\infty$.*

Here $\mathbb{R}^\infty$ is the linear space with countable Hamel basis and the strongest liner topology. For higher-dimensional Euclidean spaces $\mathbb{R}^n$ we have the following characterization connecting the topology of the group $H_c(\mathbb{R}^n)$ with the topology of the group $H_\partial(I^n)$ of homeomorphisms of the n-cube I^n that do not move the points of the boundary of I^n.

Theorem 2. *The homeomorphism group $H_c(\mathbb{R}^n)$ is an $l_2 \times \mathbb{R}^\infty$-manifold if and only if the group $H_\partial(I^n)$ is an ANR. For $n \leq 2$ the group $H_c(\mathbb{R}^n)$ is homeomorphic to $l_2 \times \mathbb{R}^\infty$.*

It is an old open problem if the homeomorphism group $H_\partial(I^n)$ is an AR for $n \geq 3$. However it is known (and easily seen) that this group is contractible.