Electron motion and ruled surfaces.

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The motion of an electron (point charge) and its emission of an electromagnetic field have been considered keeping within the bounds of classical electrodynamics. Owing to the specific properties of the emitted field, we were able to find a natural correlation between the emission and the ruled surfaces characterizing this emission in different directions. In classical electrodynamics the emission is described by two vector fields - electric E_{emit} and magnetic B_{emit}. They are orthogonal to each other and to the direction of emission. The fields E_{emit} and B_{emit} have different intensities and vectors in different directions and are described by the formulas used in our study (see [1], Eq.(19.1)). Besides, polarization is also an important property of the emission.

We consider the simplest case of the motion of a charge in a constant magnetic field. It is well known that under the influence of the constant magnetic field and the Lorentz force, the charge moves along a straight line, or a circle, or a helix.

Here we consider a helical motion. O.A.Goncharova [2] introduced standard ruled surfaces Φ_i in n-dimensional Euclidean space. In these surfaces the directing curve γ has constant non-zero curvatures $k_i \quad (i = 1, ..., n-1)$ and the straight -line generator is along the basic vector ξ_i of the natural γ-frame. For n=3 these surfaces are Φ_1 (developable helicoid), Φ_2 (helicoid) and Φ_3 (a new surface).

Let us consider the emission along a certain basic vector ξ_l at a distance ρ from the electron. The vectors E_{emit} and B_{emit} are parallel to the other basic vectors $\xi_i, \quad \xi_j, \quad l \neq i \neq j \neq l$. Translate the vectors $E_{emit}(\rho\xi_l)$ in such a way their beginnings are at the point of electron. Then their end points will be situated in the straight line directed along the basic vector ξ_i. On varying ρ, the end of the vector E_{emit} will move along the straight line advancing its or half length.

In the curse of the electron motion this straight line describes the surface Φ_i. A similar procedure can be used to construct a surface with the help of B_{emit}. Omitting the notation *emit* we can write down

$$E(\rho\xi_1) = -\frac{\sigma_1\xi_2}{\rho}, \quad E(\rho\xi_2) = -\frac{\sigma_2\xi_1}{\rho c}, \quad E(\rho\xi_3) = -\frac{\sigma_3\xi_3}{\rho},$$

$$B(\rho\xi_1) = -\frac{\sigma_1\xi_3}{\rho c}, \quad B(\rho\xi_2) = \frac{\sigma_2\xi_3}{\rho c^2}, \quad B(\rho\xi_3) = \frac{\sigma_3\xi_1}{\rho c}.$$

Here σ_i is the constant determined by the universal physical constants, the electron trajectory curvature and the electron velocity; the constant c is the velocity of light. Correspondingly, we obtain the following standard ruled surfaces:

$$\Phi_2, \quad \Phi_1, \quad \Phi_2,$$

$$\Phi_3, \quad \Phi_3, \quad \Phi_1.$$

The natural ruled surfaces appear while considering the directions along which the emission is zero. These directions are found in the osculating plane. They are symmetrical with respect to the vector ξ_1 and make an constant angle ϕ with it in the course of the helical motion of the electron. We call ϕ a zero-emission angle. The electron under consideration can be denoted as Q_1. If we draw a straight line through each point of the electron trajectory towards zero emission, we can obtain a certain ruled surface, which we identify as "a zero-emission surface". The are two surface of this type$\Psi_i, \quad i = 1, 2$. The directions of the zero emission are in the plane of the vectors ξ_1 and ξ_2. For this reason *the trajectory of the electron motion on the surface Φ_i is an asymptotic curve. This conclusion is true not only for electron motion in the constant magnetic field, but in the general case as well.*

Let us consider the question whether there exists another electron Q_2 whose zero-emission surface coincides with that of the electron Q_1. In our study [3] we proved the following theorem

Theorem. *For an electron moving in a constant magnetic field on each surface of zero emission there exists one and only one second electron so that during the motion they fall simultaneously on the common straight line of zero emission.*

The second electron can be called "conjugate"to the first electron. Note the following interesting property of the surface of zero emission: its striction curve is geodesic and takes the medial position between the specified asymptotic curves - the trajectories of the electrons Q_1 and Q_2. This curve is also helical.

References

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3. Y.A.Aminov. "On physical interpretation of some ruled surfaces in E^3 with the help of motion of point charge ", Matem. Sbornik, 2006, v. 197, N 12, p. 3-10.