Mathematical model of influenza

B. Kempińska-Mirosławska, B. Przeradzki

Medical University of Lodz, Institute of Mathematics LUT

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- Proposal of an appropriate strategy of vaccinations.

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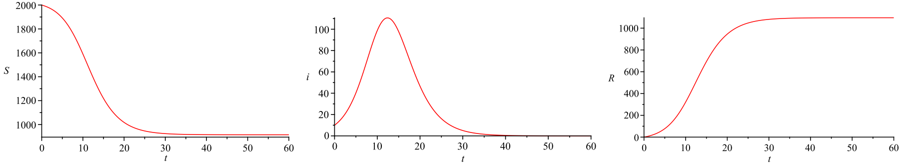

tu: $\beta = 5 \cdot 10^{-4}$, $\alpha = 0.7$, $S(0) = 2000$, $I(0) = 10$, $R(0) = 0$.

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$\delta = 0.071$ – reduction of infectiveness for asymptomatic infections,
$p = 0.3 - 0.6$ – probability of developing symptoms.

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For SLIAR: $\mathcal{R}_0 = \beta S(0)\left(\frac{p}{\alpha_I} + \frac{\delta(1-p)}{\alpha_A}\right)$

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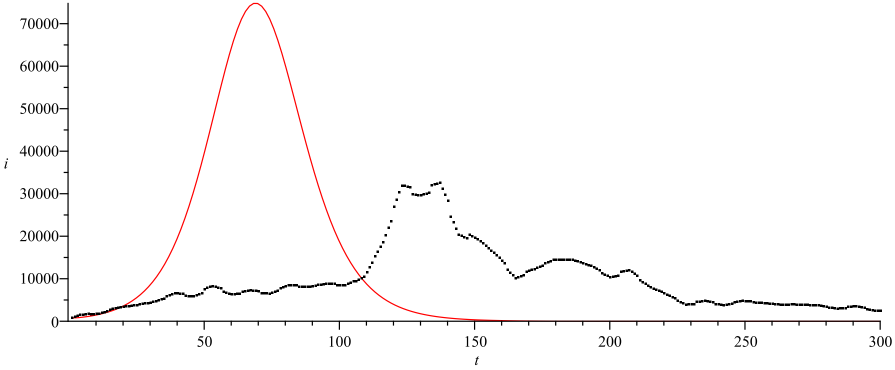

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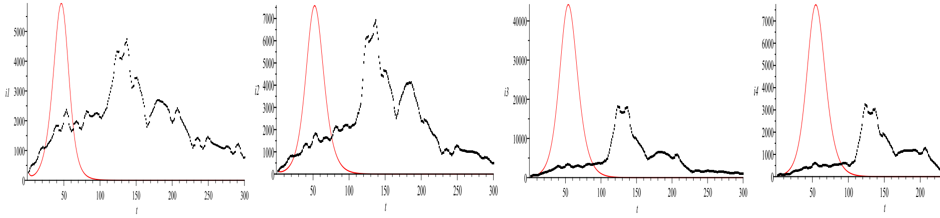

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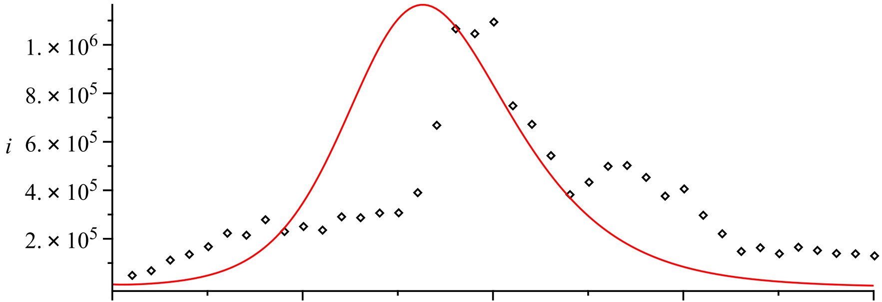

Thanks for your attention!