Mathematical model of influenza

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Goals

- To construct a model of epidemic of influenza fitting to Poland,
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- Proposal of an appropriate strategy of vaccinations.
SIR model

$S$ – number of susceptibles, $I$ – number of infectives, $R$ – number of recovered.
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$tu$: $\beta = 5 \cdot 10^{-4}$, $\alpha = 0.7$, $S(0) = 2000$, $I(0) = 10$, $R(0) = 0$. 
SLIAR model

\[ S \] – susceptibles, \( L \) – latent, noninvasion stage, \( I \) – infectives symptomatic, \( A \) – infectives asymptomatic, \( R \) – recovered.
SLIAR model


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\begin{align*}
S' &= -\beta S(I + \delta A) \\
L' &= \beta S(I + \delta A) - \alpha_L L \\
l' &= p \cdot \alpha_L L - \alpha_I I \\
A' &= (1 - p) \cdot \alpha_L L - \alpha_A A \\
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Coefficients $\alpha_L$, $\alpha_I$ and $\alpha_A$ are inverses of periods of different stages of infection: 1.25 day, 2.85 day and 4.1, day respectively,
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Coefficients $\alpha_L$, $\alpha_I$ and $\alpha_A$ are inverses of periods of different stages of infection: 1.25 day, 2.85 day and 4.1, day respectively, $\delta = 0.071$ – reduction of infectiveness for asymptomatic infections, $p = 0.3 - 0.6$ – probability of developing symptoms.
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Basic reproduction number

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For SLIAR: \( R_0 = \beta S(0) \left( \frac{p}{\alpha_I} + \frac{\delta(1-p)}{\alpha_A} \right) \)
Transmission rate

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Age structure

Epidemiological data are grouped for age intervals: 0 – 4, 5 – 14, 15 – 64 i 65 +. Improved model:
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S_j' &= -\beta_j \frac{S_j}{N} \left( (I_1 + \delta_1 A_1) N_1 + \ldots + (I_4 + \delta_4 A_4) N_4 \right) \\
L_j' &= \beta_j \frac{S_j}{N} \left( (I_1 + \delta_1 A_1) N_1 + \ldots + (I_4 + \delta_4 A_4) N_4 \right) - \mu L_j L_j \\
l_j' &= p_j \mu L_j L_j - \mu l_j l_j \\
A_j' &= (1 - p_j) \mu L_j L_j - \mu A_j A_j \\
R_j' &= \mu A_j A_j + \mu l_j l_j,
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$j = 1, 2, 3, 4.$
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Immunological memory

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S'_j &= -\beta_j S_j (I + \delta A) \\
L' &= \sum_{j=0}^{5} \beta_j S_j ((I + \delta A) - \mu_L L) \\
I' &= p\mu_L L - \mu_I I \\
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Thanks for your attention!