V Conference

GEOMETRY AND TOPOLOGY OF MANIFOLDS

Under the auspices of Prof. Jan Krysiński,
Rector of the Technical University of Łódź

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Contents

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TITLES OF LECTURES

1. **Alekseevsky, Dmitri**, Geometry of quaternionic and para–quaternionic CR manifolds
2. **Bajguz, Wiesław & Sliżewska, Aneta**, On hybrid sets – new mathematics?
3. **Balcerzak, Bogdan & Kubarski, Jan**, Unification of Crainic and Kubarski secondary characteristic classes for Lie algebroids
4. **Belko, Ivan,** The fundamental form on a Lie groupoid of diffeomorphisms
5. **Bogdanovich, Sergey A.**, Hypercomplex structures on tangent bundles
6. **Borak, Ewa & Kwaśniewski, Andrzej Krzysztof**, Extended finite operator calculus – an example of algebraization of analysis
7. **Deszcz, Ryszard,** On Roter type manifolds
8. **Ermolitski, Alexander A.**, Deformations of distributions on Riemannian manifolds
9. **Głogowska, Małgorzata,** On quasi-Einstein Cartan type hypersurfaces
10. **Haller, Stefan**, Harmonic cohomology of symplectic manifolds
11. **Hansoul, Sarah**, Geometric interpretation of cohomology classes and application to the case of smooth manifolds
12. **Homolya, Szilvia**, Geodesics of two–step nilpotent Lie groups
13. **Hotloś, Marian**, On certain Ricci-pseudosymmetric hypersurfaces in space forms
14. **Iwase, Norio**, Lusternik–Schnirelmann category of Lie groups
15. **Kock, Anders**, Second neighbourhood of the diagonal, and a foundation for conformal geometry
16. **Kolář, Ivan**, Flow prolongation of projectable tangent valued forms
17. **Krot, Ewa**, Remarks on Fibonomial Calculus
18. **Kucharski, Zygfryd,** The Nielsen Number respect to submanifold
19. **Kurek, Jan, Mikulski, Włodzimierz,** Symplectic structures of the tangent bundles of symplectic and cosymplectic manifolds
20. **Lecomte**, **Pierre,** Martin Bordemann's proof of the existence of projectively equivariant quantizations

21. **Łuczyszyn Dorota,** On the Bochner curvature of para–Käehlerian manifolds

22. **Mishchenko, Alexandr & Kubarski, Jan** Transitive Lie algebroids: spectral sequences and signature

23. **Mishchenko, Tatiana**, Goals, Content and Framework of the Educational Standard of Russia in the School Mathematics

24. **Mormul, Piotr**, Geometric singularity classes for special k-flags ($k \geqslant 2$) of arbitrary length

25. **Mykytyuk, Ihor**, Invariant Hyper-Kähler Structures on the Cotangent Bundles of Hermitian Symmetric Spaces

26. **Nagy, Péter Tibor**, The classification of compact smooth Bol loops

27. **Nazaykinskiy, Vladimir,** Toeplitz Representations for Operators on Singular Manifolds

28. **Nguiffo Boyom, Michel**, Quadratization of Lie algebroids

29. **Olszak, Zbigniew,** On almost complex structures with Norden metrics on tangent bundles

30. **Plachta, Leonid**, Essential tori in link complements in standard positions: geometric and combinatorial aspects

31. **Poncin, Norbert & Grabowski, Janusz**, Lie algebras of differential operators

32. **Pradines, Jean**, Lie Groupoids as generalized atlasses for spaces of leaves

33. **Rybicki, Tomasz**, Some general remarks on foliated structures

34. **Savin, Anton**, Elliptic operators on singular manifolds and K–homology

35. **Sawicz, Katarzyna**, On some class of hypersurfaces with pseudosymmetric Weyl tensor

36. **Sinaiskii, Eugenij**, Translation continuous functionals on CB($\mathbb{R}$) and their supports

37. **Skopenkov, Arkadiy**, The Whitehead torus, the Hudson-Habegger invariant and classification of embeddings $S^1 \times S^3 \to \mathbb{R}^7$

38. **Sternin, Boris**, Elliptic Operators on Manifolds with Nonisolated Singularities

39. **Szenthe, János**, On spherically symmetric space-time

40. **Tosun, Murat**, Generalized Space–Like Ruled Surfaces of the Minkowski Space $\mathbb{R}^n_1$

41. **Tulczyjew, Włodzimierz**, Analytical Mechanics with Discontinuities and Disipation

42. **Urbański, Paweł**, AV-geometry and classical mechanics

43. **Walas, Witold & Mishchenko, Alexandr,** Lie algebroids over S^1 and monodromy

44. **Waliszewski, Włodzimierz**, Quasi-polyhedrons

45. **Yu, Yanlin**, Heat Kernel Quantization

46. **Zhang, Weiping**, Circle actions and $\mathbf{Z}/k-$manifolds

Programme

MONDAY, 28 April 2003

9.00	Opening conference
9.10 – 10.10	Opening lecture: **J. Pradines,** **Lie Groupoids as generalized atlasses for spaces of leaves**
10.25 – 11.10	Plenary lecture: **I. Belko,** **The fundamental form on a Lie groupoid of diffeomorphisms**
11.10 – 11.40	Coffee break
11.40 – 12.20	**M. Nguiffo Boyom,** **Quadratization of Lie algebroids**
12.30 – 13.00	**T. Rybicki,** **Some general remarks on foliated structures**
13.00 – 15.00	**LUNCH**
15.00 – 15.30	**A. Savin,** **Elliptic operators on singular manifolds and K–homology**
15.40 – 16.20	**B. Sternin,** **Elliptic operators on manifolds with nonisolated singularities**
16.20 – 16.40	Coffee break
16.40 – 17.10	**S. Honsul,** **Geometric interpretation of cohomology classes and application to the case of smooth manifolds**
17.20 – 17.50	**A. Skopenkov,** **The Whitehead torus, the Hudson-Habegger invariant and classification of embeddings** $S^1 \times S^3 \to \mathbb{R}^7$
18.30 – to Saturday	**SESSION OF POSTERS, SESSION OF POSTERS OF VIRTUAL PARTICIPANTS**

TUESDAY, 29 April 2003

9.00 – 10.00	Plenary lecture: **A.S. Mishchenko (coauthor J. Kubarski)** **Transitive Lie algebroids: spectral sequences and signature**
10.10 – 10.55	Plenary lecture: **L. Plachta,** **Essential tori in link complements in standard positions: geometric and combinatorial aspects**
10.55 – 11.25	Coffee break
11.25 – 12.10	Plenary lecture: **N. Poncin (coauthor J.Grabowski),** **Lie algebras of differential operators**
12.20 – 13.00	**I. Kolář,** **Flow prolongation of projectable tangent valued forms**
13.00 – 15.00	**LUNCH**
15.00 – 15.30	**A. Ermolitski,** **Deformations of distributions on Riemannian manifolds**
15.40 – 16.10	**P. Urbański,** **AV-geometry and classical mechanics**
16.10 – 16.40	Coffee break
16.40 – 17.10	**S. Homolya,** **Submersions on nilmanifolds and their geodesics**
17.20 – 17.50	**Z. Kucharski,** **The Nielsen Number respect to submanifold**
18.00 – 18.30	**E.Sinaiskii,** **Translation continuous functionals on CB(G) and their support**
20.00	CONCERT OF LOCAL BAND OF MUSICIANS

WEDNESDAY, 30 April 2003

THE DAY OF TOURISM

19.30 SUPPER BY BONFIRE

THURSDAY, 1 May 2003

9.00 – 10.00	Plenary lecture: **D. Alekseevsky** **Geometry of quaternionic and para-quaternionic CR manifolds**
10.10 – 10.55	Plenary lecture: **P. Lecomte,** **Bordemann's proof of the existence of projectively equivariant quantizations**
10.55 – 11.25	Coffee break
11.25 – 12.05	Plenary lecture: **J. Kurek (coauthor W. Mikulski),** **Symplectic structures of the tangent bundles of symplectic and cosymplectic manifolds**
12.15 – 12.55	**S.Haller,** **Harmonic cohomology of symplectic manifolds**
13.00 – 15.00	**LUNCH**
15.00 – 15.30	**Z. Olszak,** **On almost complex structures with Norden metrics on tangent bundles**
15.40 – 16.10	**S. Bogdanovich,** **Hypercomplex structure on tangent bundles**
16.10 – 16.40	Coffee break
16.40 – 17.10	**D. Łuczyszyn,** **On the Bochner curvature of para–Käehlerian manifolds**
17.20 – 17.50	**I. Mykytyuk,** **Invariant Hyper-Kähler Structures on the Cotangent Bundles of Hermitian Symmetric Spaces**
18.00 – 18.30	**P. Mormul,** **Geometric singularity classes for special k-flags ($k \geqslant 2$) of arbitrary length**
19.30	GALA PARTY

FRIDAY, 2 May 2003

9.00 – 10.00	Plenary lecture: **A.Kock** **Second neighbourhood of the diagonal, and the foundation for conformal geometry**
10.10 – 10.40	Plenary lecture: **P. Nagy,** **The classification of compact smooth Bol loops**
10.40 – 11.10	Coffee break
11.10 – 11.40	**W. Waliszewski,** **Quasi–polyhedrons**
11.50 – 12.20	**W. Bajguz (coauthor A.Sliżewska),** **On hybrid sets – new mathematics?**
12.30 – 13.00	**T. Mishchenko** **Goals, content and framework of the educational standard of Russia in the school mathematics**
13.00 – 15.00	**LUNCH**
15.00 – 15.30	**E. Borak (coauthor A. K. Kwaśniewski),** **Extended finite operator calculus - an example of algebraization of analysis**
15.40 – 16.10	**E.Krot,** **Remarks on Fibonomial Calculus**
16.10 – 16.40	Coffee break
16.40 – 17.10	**R. Deszcz,** **On Roter type manifolds**
17.20 – 17.50	**M. Głogowska,** **On quasi-Einstein Cartan type hypersurfaces**
18.00 – 18.30	**M. Hotloś,** **On certain Ricci-pseudosymmetric hypersurfaces in space forms**
19.30	**PROBLEMS SESSION**

SATURDAY, 3 May 2003

9.00 – 9.30	**B. Balcerzak (coauthor J. Kubarski),** **Unification of Crainic and Kubarski secondary characteristic classes for Lie algebroids**
9.30 – 10.00	THE LAST COFFEE BREAK
10.00 – 11.00	Plenary lecture: **W. Tulczyjew,** **Analytical Mechanics with Discontinuities and Disipation**
11.00	CLOSING CEREMONY

ABSTRACTS

Geometry of quaternionic and para-quaternionic CR manifolds

Dmitri Alekseevsky

Department of Mathematics, Hull University, UK

Notions of quaternionic and para-quaternionic CR structures on a $(4n+3)$ - dimension manifold M are defined as a triple of 1-forms $\omega = (\omega_1, \omega_2, \omega_3)$ which satisfy some conditions. We associate with such a structure a pseudo-Riemannian Einstein metric g on M and a Lie algebra $\mathfrak{a}$ of its Killing fieleds , isomorphic to $sp(1)$ in quaternionic case and $sp(1, \mathbb{R})$ in para-quaternionic case. If the metric g is positively defined, then a quaternionic CR structure is equivalent to a Sasakian 3-structure. We give examples of homogeneous manifolds with invariant quaternionic and para-quaternionic CR structure and describe a reduction method, which allows to construct non-homogeneous quaternionic and para-quaternionic CR structures starting from a manifold with such a structure which has a symmetry Lie group. It is shown also that a cone over a manifold M with (para)-quaternionic CR structure carries a (para)-hyperKaehler structure and the quotient M/A of M by the Lie group A of isometries , generated by $\mathfrak{a}$, carries a (para)-quaternionic Kaehler structure (under assumption that the group A is defined and acts properly on M.)

This is a joint work with Y. Kamishima.

The fundamental form on a Lie groupoid of diffeomorphisms

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The Ehresmann method is an effective method of study of geometrical structures on differentiable manifolds. It is based on the theory of jets and Lie groupoids and it is the development of E. Cartan ideas. The Lie groupoid $\Pi^k(B)$ of k-jets of local diffeomorphisms of differentiable manifold B is essential in the Ehresmann method. This Lie groupoid admits itself many special structures. We mark the following structures among the last ones:

(i) the canonical morphisms of Lie groupoids

$$\pi^k_{k-1} : \Pi^k(B) \longrightarrow \Pi^{k-1}(B);$$

(ii) the representation of Lie algebroid $A\Pi^k(B)$ as a Lie algebroid J^kTB of k-jets of vector fields on B;

(iii) the truncated bracket $A\Pi^k(B)\Lambda A\Pi^k(B) \longrightarrow A\Pi^{k-1}(B)$, which is a vector bundle morphism;

(iv) the representation of Lie groupoid $\Pi^k(B)$ as a Lie groupoid of a vector bundle isomorphisms preserving the truncated bracket;

(v) the fundamental form on $\Pi^k(B)$ with values in the Lie algebroid $A\Pi^{k-1}(B)$.

The called structures are generalizations of the similar structures on the frame bundles of higher order, which has been studying by V.Guillemin and S. Sternberg, P. Liebermann, P. Molino, Ngo van Que, D. Alekseevsky and P.Michor and others.

Our goal is an exposition of the basic theory of G-structures with an emphasis on Ehresmann method. The Lie groupoid $\Pi^k(B)$ consists of k-jets of local diffeomorphism of B. The Lie groupoid $\Pi^1(B)$ is a Lie groupoid of linear isomorphisms of the tangent bundle TB. The maps α and β are the source and the target maps. For any $x \in B$ α-leaf $\alpha^{-1}(x)$ is the principal bundle $\Pi^k(B)_x(B, \beta, G^x_x)$ of higher order frames. The right translation of $\Pi^k(B)$ is an isomorphism of principale bundles. For any diffeomorphism φ of B the map $j^k\varphi$ defines an admissible section for α-projection. This section defines itself a left translation φ^k of Lie groupoid $\Pi^k(B)$. Let $A\Pi^k(B)$ be a Lie algebroid of $\Pi^k(B)$. This Lie algebroid is isomorphic to Lie algebroid J^kTB of k-jets of vector fields. The section $j^kX, X \in \Gamma TB$ defines a vector field $X^{(k)} \in \Gamma A\Pi^k(B)$ with a current

$$exp_tX^{(k)}(j^k_x\psi) = j^k_x(exp_tX \circ \psi).$$

The truncated bracket on J^kTB is defined in such a manner. The Lie bracket of Lie algebroid

$$\Gamma J^{k-1}TB\Lambda\Gamma J^{k-1}TB \longrightarrow \Gamma J^{k-1}TB$$

is on operator differential when any of its arguments is fixed. That's why it defines a vector bundles morphism

$$J^kTB\Lambda J^kTB \longrightarrow J^{k-1}TB.$$

This morphism becomes the truncated bracket of Lie algebroid $A\Pi^k(B)$. The truncated bracket is very important for the next theorem.

Theorem 1. *The Lie groupoid $\Pi^k(B)$ is isomorphed to Lie groupoid of vector bundle isomorphisms of $A\Pi^{k-1}(B)$, which conserve the truncated bracket.*

There fore any element $u \in \Pi^k(B)^y_x$ can be considered as linear isomorphism

$$u : A\Pi^{(k-1)}(B)_x \longrightarrow A\Pi^{(k-1)}(B)_y.$$

The fundamental form ϑ on $\Pi^k(B)$ is an α-vertical form with values in the Lie algebroid $A\Pi^{(k-1)}(B)$. Let $p \in T^\alpha_u\Pi^k(B)$ is an α-vertical tangent vector. Then

$$\vartheta_u(p) = u^{-1} \circ (\pi^k_{k-1})_*(p).$$

The form ϑ characterizes the continuations of basic diffeomorphisms.

Theorem 2. *Let ψ be a local diffeomorphism of $\Pi^k(B)$, which conserves the α-leaves and the fundamental form. Then ψ coincides with a local left translation of $\Pi^k(B)$.*

Hypercomplex structures on tangent bundles

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With help of any metric connection $\widetilde{\nabla}$ on an almost Hermitian manifold M we can construct by the defined way an almost Hermitian hypercomplex structure on the tangent bundle TM. This structure includes two basic anticommutative almost Hermitian structures for which introduced by the second author the fundamental tensor fields h^1 and h^2 are computed. It allows to consider various classes of almost Hermitian hypercomplex structures on TM.

1. Introduction.

Let (M, J, g) be an almost Hermitian manifold i.e. $J^2 = -I$ and $g(JX, JY) = g(X, Y)$ for $X, Y \in \chi(M)$, where g is a fixed Riemannian metric on M.

For the Riemannian connection ∇ the canonical connection $\overline{\nabla}$ of the pair (J, g) [1] is defined by the formula

(1) $\quad \overline{\nabla}_X Y = \frac{1}{2}(\nabla_X Y - J\nabla_X JY) = \nabla_X Y + \frac{1}{2}\nabla_X(J)JY, \qquad X, Y \in \chi(M).$

The tensor field $h = \nabla - \overline{\nabla}$ is called the second fundamental tensor field of the pair (J, g) [1], in particular, we have $\overline{\nabla} g = 0$, $\overline{\nabla} J = 0$ and

(2) $\quad h_X Y = -\frac{1}{2}\nabla_X(J)JY = \frac{1}{2}(\nabla_X Y + J\nabla_X JY),$

(3) $\quad h_{XYZ} = g(h_X Y, Z) = -h_{XZY}, \qquad X, Y \in \chi(M).$

The classification given in [3] has been rewritten in terms of the tensor field h in [1].

Further, an almost Hermitian hypercomplex structure (aHhs) consists of (J_1, J_2, J_3, g), where $J_i^2 = -I$, $J_1J_2 = -J_2J_1 = J_3$, $g(J_iX, J_iY) = g(X, Y)$, $i = 1, 2, 3$. For any Riemannian metric $\widetilde{g}$ such a metric g can be defined by the formula

$$g(X, Y) = \tfrac{1}{4}\left(\widetilde{g}(X, Y) + \widetilde{g}(J_1X, J_1Y) + \widetilde{g}(J_2X, J_2Y) + \widetilde{g}(J_3X, J_3Y)\right).$$

If ∇ is the Riemannian connection of the metric g then the canonical connection $\overline{\nabla}$ of the aHhs has the following form

(4) $\quad \overline{\nabla}_X Y = \frac{1}{4}\left(\nabla_X Y - J_1\nabla_X J_1Y - J_2\nabla_X J_2Y - J_3\nabla_X J_3Y\right)$

and $\overline{\nabla} g = 0$, $\overline{\nabla} J_i = 0$ for $i = 1, 2, 3$.

Proposition. *Let (M, J_1, g) be a Kaehlerian structure i.e. $\nabla J_1 = 0$ on M then the connection given by (4) coincides with those defined by (1) for (M, J_2, g) and (M, J_3, g). In particular, the second fundamental tensor fields of (M, J_2, g) and (M, J_3, g) are the same.*

Proof follow from (4) and (1) with help of condition $\nabla J_1 = 0$.

Theorem. *A vector field X is an infinitesimal isometry and an affine transformation with respect to $\overline{\nabla}$ defined by (4) if and only if $L_X g = 0$ and $L_X h = 0$, where $h = \nabla - \overline{\nabla}$, L is the Lie differentiation with respect to X.*

2. Hypercomplex structures on tangent bundles.

Let (M,J,g) be an almost Hermitian manifold and TM be its tangent bundle. For a metric connection $\widetilde{\nabla}$ $\left(\widetilde{\nabla} g=0\right)$ we consider the connection map $\widetilde{K}$ of $\widetilde{\nabla}$ [2], defined by the formula

$$\widetilde{\nabla}_X Z=\widetilde{K} Z_* X,$$

where Z is considered as a map from M into TM and we means by the right side the vector field on M assigning to $p\in M$ the vector $\widetilde{K} Z_* X_p\in M_p$. If $U\in TM$, we denote by H_U the kernel of $\widetilde{K}_{|TM_U}$ and this $2n$-dimensional subspace of TM_U is called the horizontal subspace of TM_U. Let π denote the natural projection of TM onto M then π_* is a C^∞-map of TTM onto TM. If $U\in TM$, we denote by V_U the kernel of $\pi_{*|TM_U}$ and this $2n$-dimensional subspace of TM_U is called the vertical subspace of TM_U ($\dim TM_U=2\dim M=4n$). The following maps are isomorphisms of corresponding vector spaces ($p=\pi(U)$).

$$\pi_{*|TM_U}: H_U\longrightarrow M_p,\qquad \widetilde{K}_{|TM_U}: V_U\longrightarrow M_p$$

and we have $TM_U=H_U\oplus V_U$.

If $X\in\chi(M)$ then there exists exactly one vector field on TM called the "horizontal lift" (resp. "vertical lift") of X and denoted by X^h (resp. X^v) such that for all $U\in TM$:

$$\pi_* X^h_U=X_{\pi(U)},\qquad \pi_* X^v_U=O_{\pi(U)};\qquad \widetilde{K} X^h_U=O_{\pi(U)},\qquad \widetilde{K} X^v_U=X_{\pi(U)}.$$

Let $\widetilde{R}$ be the curvature tensor field of $\widetilde{\nabla}$ then following [2] we have

$$[X^v,Y^v]=0,\qquad \left[X^h,Y^v\right]=(\widetilde{\nabla}_X Y)^v,\qquad \pi_*([X^h,Y^h]_U)=[X,Y],$$
$$\widetilde{K}([X^h,Y^h]_U)=\widetilde{R}(X,Y)U.$$

For vector fields $\overline{X}=X^h\oplus X^v$ and $\overline{Y}=Y^h\oplus Y^v$ on TM the natural Riemannian metric $<,>$ is defined on TM by the formula

$$<\overline{X},\overline{Y}>=g(\pi_*\overline{X},\pi_*\overline{Y})+g(\widetilde{K}\overline{X},\widetilde{K}\overline{Y}).$$

It is clear that the subspaces H_U and V_U are orthogonal with respect to $<,>$.

I). We define a tensor field J_1 on TM by the equalities

$$J_1X^h=X^v,\qquad J_1X^v=-X^h,\qquad X\in\chi(M).$$

It is easy to verify that $(TM,J_1,<,>)$ is an almost Hermitian manifold.

Remark. *This construction uses only the Riemannian metric g and does not depend on the almost complex structure J.*

Let h^1 be the second fundamental tensor field of the pair $(J_1,<,>)$, see (2), (3). We have obtained the following cases for the tensor field h^1 assuming all the vector fields to be orthonormal

1.1$^{\mathbf{0}}$) $\quad h^1_{X^hY^hZ^h}=\frac{1}{2}\left(g(\nabla_X Y,Z)-g(\widetilde{\nabla}_X Y,Z)\right);$

2.1$^{\mathbf{0}}$) $\quad h^1_{X^hY^hZ^v}=-\frac{1}{4}\left(g(\widetilde{R}(X,Y)Z,U)+g(\widetilde{R}(Z,X)Y,U)\right);$

3.1⁰) $h^1_{X^hY^vZ^h} = -\frac{1}{4}\Big(g(\widetilde{R}(Z,X)Y,U) + g(\widetilde{R}(X,Y)Z,U)\Big);$
4.1⁰) $h^1_{X^vY^hZ^h} = -\frac{1}{4}g(\widetilde{R}(Z,Y)X,U);$
5.1⁰) $h^1_{X^vY^vZ^v} = \frac{1}{4}g(\widetilde{R}(Z,Y)X,U);$
6.1⁰) $h^1_{X^vY^vZ^h} = 0;$
7.1⁰) $h^1_{X^vY^hZ^v} = 0;$
8.1⁰) $h^1_{X^hY^vZ^v} = \frac{1}{2}\Big(g(\widetilde{\nabla}_XY,Z) - g(\nabla_XY,Z)\Big).$

Thus, the tensor field h^1 (class of the structure $(J_1,<,>)$) strongly depends on the connection $\widetilde{\nabla}$.

II). We define a tensor field J_2 on TM assuming

$$J_2X^h = (JX)^h, \qquad J_2X^v = -(JX)^v, \qquad X \in \chi(M).$$

One can verify that $(TM, J_1, J_2, J_3 = J_1J_2, <,>)$ is an almost Hermitian hypercomplex manifold.

Let h^2 be the second fundamental tensor field of the pair $(J_2,<,>)$, see (2), (3). Assuming all the vector fields to be orthonormal we have got

1.2⁰) $h^2_{X^hY^hZ^h} = h_{XYZ};$
2.2⁰) $h^2_{X^hY^hZ^v} = -\frac{1}{4}\Big(g(\widetilde{R}(X,Y)Z,U) + g(\widetilde{R}(X,JY)JZ,U)\Big);$
3.2⁰) $h^2_{X^hY^vZ^h} = \frac{1}{4}\Big(g(\widetilde{R}(X,Z)Y,U) + g(\widetilde{R}(X,JZ)JY,U)\Big);$
4.2⁰) $h^2_{X^vY^hZ^h} = -\frac{1}{4}\Big(g(\widetilde{R}(Z,Y)X,U) - g(\widetilde{R}(JZ,JY)X,U)\Big);$
5.2⁰) $h^2_{X^vY^vZ^v} = 0;$
6.2⁰) $h^2_{X^vY^vZ^h} = 0;$
7.2⁰) $h^2_{X^vY^hZ^v} = 0;$
8.2⁰) $h^2_{X^hY^vZ^v} = \frac{1}{2}\Big(g(\widetilde{\nabla}_XY,Z) - g(\widetilde{\nabla}_XJY,JZ)\Big).$

It is clear that the construction of the aHhs on TM strongly depends on the connection $\widetilde{\nabla}$ and we can obtain in this vay an infinite dimensional set of aHhs.

Using the *remark* in I) and the arguments above we have got the following

Theorem. *Let (M,g) be a Riemannian manifold. Then there exists an infinite dimensional set of aHhs on TTM. This structures can be constructed by the method above.*

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On Roter type manifolds

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Abstract. We introduce the notion of Roter type manifolds. We present curvature properties of such manifolds. The main results are related to the case when, in addition, these manifolds are warped products. Suitable examples will be given.

Deformations of distributions on Riemannian manifolds

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$\mathbf{1^0}$. Let M be a connected C^∞-manifold, X be a nonsingular vector field on M, $L(M)$ be the set of all linear frames at all points of M with the structure group $GL(n, \mathbf{R})$. We can take a Riemannian metric g on the manifold M and for a point $p \in M$ we consider open geodesic balls $B\left(p; \frac{R}{2}\right) \subset B(p; R) \subset U$, where U is a coordinate neighborhood on M. There exists such a coordinate system $(x_1, x_2, ..., x_n)$ on U, where $n = \dim M$, that $X = \frac{\partial}{\partial x_1}$ on U.

Further, let we have a vector field Y $(Y_p = X_p)$ and such a coordinate system $(y_1, y_2, ..., y_n)$ on U that $Y = \frac{\partial}{\partial y_1}$ on U and $\frac{\partial}{\partial y_i}_{|_p} = \frac{\partial}{\partial x_i}_{|_p}$, $i = \overline{1, n}$. For any point $x \in M$ we get

$$\left(\frac{\partial}{\partial y_1}, \frac{\partial}{\partial y_2}, ..., \frac{\partial}{\partial y_n}\right)_x = \nu(x)\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, ..., \frac{\partial}{\partial x_n}\right)_x,$$

where $\nu(x) \in GL(n, \mathbf{R})$.

Thus, we have obtained the mapping

$$\nu : U \to GL(n, \mathbf{R}) : x \to \nu(x); \qquad \nu(p) = e \in GL(n, \mathbf{R}).$$

If $\exp_p$ is the exponential mapping of the Riemannian connection ∇ then $\exp_p^{-1}$ is defined on the closed ball $\overline{B}(p; R)$ and we can define the following nonsingular vector field Z on M

$$(1) \qquad Z = \begin{cases} X_x, \quad x \in M \setminus \overline{B}(p; R); \\ \left(\nu\left[\exp_p\left(\frac{2t-R}{t}\exp_p^{-1}(x)\right)\right]\right)^{-1} Y_x, \quad x \in \overline{B}(p; R) \setminus \overline{B}(p; \frac{R}{2}); \\ Y_x, \quad x \in \overline{B}(p; \frac{R}{2}), \end{cases}$$

where $\exp_p^{-1}(x) = t\xi, \qquad \|\xi\| = 1$.

Definition. *The vector field Z is called a deformation of the vector field X to the vector field Y in a neighborhood of $p \in M$ by the formula (1).*

In particular, if $M = \mathbf{E}^n$ we can take such a vector field Y that its integral curves are simply intervals of straight lines.

$\mathbf{2^0}$. Let $x \to V(x)$ be a differentiable distribution on the manifold M and for any $x \in M$ $\dim V(x) = k$. For each point $p \in M$ there exist a coordinate neighborhood $U \ni p$ and such a local cross-section s_1 over U

$$s_1 : U \to L(M) : x \to (X_{1_x}, ..., X_{k_x}, X_{k+1_x}, ..., X_{n_x}),$$

that for any $x \in U$ $\quad X_{1_x}, ..., X_{k_x} \in V(x)$. Let $x \to V'(x)$ be another distribution on U and $V'(p) = V(p)$. We can choose such a local cross-section s_2 over U

$$s_2 : U \to L(M) : x \to (Y_{1_x}, ..., Y_{k_x}, Y_{k+1_x}, ..., Y_{n_x})$$

that for any $x \in M \quad Y_{1_x}, ..., X_{k_x} \in V'(x)$ and $Y_{i_p} = X_{i_p}, \ \ i = \overline{1,n}$.
For a point $x \in U$ we define $\nu(x) \in GL(n, \mathbf{R})$ by the formula
(2) $\qquad (Y_1, ..., X_n)_x = \nu(x)\,(X_1, ..., X_n)_x$.
So, we have got the mapping

$$\nu : U \to GL(n, \mathbf{R}) : x \to \nu(x); \qquad \nu(p) = e \in GL(n, \mathbf{R}).$$

For the geodesic balls $B(p; \frac{R}{2}) \subset B(p; R) \subset U$ the distribution $\overline{V}$ of the dimension k is defined by the formula

$$(3) \qquad \overline{V} = \begin{cases} V(x), \quad x \in M \setminus \overline{B}(p;R); \\ L\left[\left(\nu\left[\exp_p\left(\frac{2t-R}{t}\exp_p^{-1}(x)\right)\right]\right)^{-1}(Y_{1_x}, ..., Y_{k_x})\right], \ \ x \in \overline{B}(p;R) \setminus \overline{B}(p; \frac{R}{2}); \\ V'(x), \quad x \in \overline{B}(p; \frac{R}{2}), \end{cases}$$

where $\exp_p^{-1}(x) = t\xi$, $\|\xi\| = 1$, $L[...]$ is the linear span of the corresponding vectors. Let $(x_1, ..., x_n)$ be such a coordinate system on U that $\frac{\partial}{\partial x_i}_{|p} = X_{i_p}, \quad i = \overline{1,n}$. Taking $Y_{i_x} = \frac{\partial}{\partial x_i}_{|x}$ on U we obtain the distribution $V'(x) = L[Y_{1_x}, ..., Y_{k_x}]$. It is obvious that $V'(x)$ is integrable on U.

Thus, we have obtained the following

Theorem. *Let $V(x)$ be a differentiable distribution on M and $p \in M$. Then, there exists such a deformation $\overline{V}(x)$ defined by the formula (2) that $\overline{V}(x)$ is integrable on some neighborhood of the point p.*

We can consider the Riemannian metric g' on U assuming $g'\left(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right) = \delta^i_j$. There exists the Riemannian metric $\overline{g}$ on M defined by the formula

$$(4) \qquad \overline{g}_x = \begin{cases} g_x, \quad x \in M \setminus \overline{B}(p;R); \\ \frac{2t-R}{R} g_x + \left(1 - \frac{2t-R}{R}\right) g'_x, \ \ x \in \overline{B}(p;R) \setminus \overline{B}(p; \frac{R}{2}); \\ g'_x, \quad x \in \overline{B}(p; \frac{R}{2}), \end{cases}$$

where $\exp_p^{-1}(x) = t\xi, \quad \|\xi\| = 1$.

We can consider the orthogonal complement $\overline{V}^{\perp}(x)$ to $\overline{V}(x)$ on M with respect to $\overline{g}$ i.e. $M_x = \overline{V}(x) \oplus \overline{V}^{\perp}(x)$ for any $x \in M$. It is evident that $\overline{V}^{\perp}(x)$ the linear span of the vectors $\frac{\partial}{\partial x_{k+1}}_{|x}, ..., \frac{\partial}{\partial x_n}_{|x}$ for each $x \in B(p; \frac{R}{2})$. For Riemannian connection $\overline{\nabla}$ of the Riemannian metric $\overline{g}$ we have $\overline{\nabla}_{\frac{\partial}{\partial x_i}} \frac{\partial}{\partial x_j} = 0$ and curvature tensor field $\overline{R}$ of $\overline{\nabla}$ vanishes.

Using for example [1] we have got the following

Theorem. *Let $V(x)$ be a differentiable distribution on (M, g) and $p \in M$. Then, there exist such deformations $\overline{V}(x)$ defined by (2) and $\overline{g}_x$ defined by (4) that the almost Riemannian product structure $M_x = \overline{V}(x) \oplus \overline{V}^{\perp}(x)$ with respect to $\overline{g}$ on M is the local Riemannian product structure on some neighborhood of the point p.*

$\mathbf{3^0}$. Let M be a $(2n+1)$-dimensional manifold and (F, ξ, η, g) be an almost contact metric structure (acms) on M, where ξ ($\|\xi\| = 1$) is a vector field on M, $F\xi = 0$, $L = L[\xi]$, $V = L^{\perp}$, $F^2 = -I$ on V, $\eta(X) = g(X, \xi)$, $X \in \chi(M)$, $g(FX, FY) = g(X, Y)$ for $X, Y \in V$. An acms defines H-structure $P(H)$ on M where $H = U(n) \times 1$.

Let $s_1 : U \to P(H) : x \to (X_{1_x}, ..., X_{n_x}, X_{n+1_x}, ..., X_{2n_x}, X_{2n+1_x})$ be a local cross-section of $P(H)$ over $U \ni p$ i.e. $FX_i = X_{n+1}$, $i = \overline{1,n}$; $FX_j = -X_{j-n}$, $j = \overline{n+1, 2n}$; $X_{2n+1} = \xi$ on U.

Further, we consider such a coordinate system $(y_1, y_2, ..., y_{2n+1})$ on U that $\frac{\partial}{\partial y_{2n+1}} = \xi$ on U and $\frac{\partial}{\partial y_i}_{|_p} = X_{i|_p}$, $i = \overline{1, 2n}$.

For any point $x \in U$ we suppose

$$\left(\frac{\partial}{\partial y_1}, \frac{\partial}{\partial y_2}, ..., \frac{\partial}{\partial y_{2n+1}}\right)_x = \nu(x)\,(X_1, ..., X_{2n+1})_x, \qquad \nu(p) = e \in GL(n, \mathbf{R}).$$

We have obtained the local cross-section $s_2 : U \to GL(n, \mathbf{R}) : x \to (Z_1, ..., Z_{2n+1})_x$ by the formula

$$(5) \qquad Z_i = \begin{cases} X_i, \quad x \in U \setminus \overline{B}(p; R); \\ \left(\nu\left[\exp_p\left(\frac{2t-R}{t}\exp_p^{-1}(x)\right)\right]\right)^{-1} \frac{\partial}{\partial y_i}_{|_x}, \;\; x \in \overline{B}(p; R) \setminus \overline{B}(p; \frac{R}{2}); \\ \frac{\partial}{\partial y_i}, \quad x \in \overline{B}(p; \frac{R}{2}). \end{cases}$$

We can define an acms on U, $\overline{\xi} = \xi$, $\overline{F}Z_i = Z_{n+i}$, $i = \overline{1,n}$; $\overline{F}Z_j = -Z_{j-n}$, $j = \overline{n+1, 2n}$, $\overline{g}(Z_i, Z_j) = \delta^i_j$, $\overline{F}\xi = 0$, $\overline{\eta}(X) = g(X, \xi)$. It is clear that the "new" acms coincides with the "old" acms on $U \setminus B(p; R)$. So, we have got the following

Theorem. *Let (F, ξ, η, g) be an acms on M and $p \in M$. Then, there exists such an acms $\left(\overline{F}, \overline{\xi}, \overline{\eta}, \overline{g}\right)$ on M defined with help of formula (5) that in some neighborhood of the point p M is a local Riemannian product $U_1 \times \overline{U}$, $\dim U_1 = 1$ and $\overline{U}$ is a Kaehlerian manifold.*

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On quasi-Einstein Cartan type hypersurfaces

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Abstract. We introduce the notion of Cartan type hypersurfaces. We present curvature properties of such hypersurfaces in semi-Riemannian space forms. The main results are related to the case when, in addition, these hypersurfaces are quasi-Einstein. Suitable examples will be given.

Harmonic cohomology of symplectic manifolds

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Abstract. On symplectic manifolds one can speak of (symplectically) harmonic differential forms and thus of harmonic cohomology classes. These were introduced by Brylinski who also asked whether every cohomology class had a harmonic representative ('symplectic Hodge theory'). This turned out to be false in general. According to a theorem of Mathieu this is the case iff the manifold satisfies the 'Hard Lefschetz Theorem'. We will see that one can explicitly compute the harmonic cohomology of a symplectic manifold in terms of its cohomology ring and the cohomology class of the symplectic form. Similar methods can be used to show that a class of symplectic manifolds (satisfying a weakened Lefschetz condition) has the c–splitting property. That is every Hamiltonian fiber bundle with such a manifold as typical fiber c–splits, i.e. the cohomology of the total space is additively the same as the cohomology of the product (trivial fiber bundle).

Submersions on nilmanifolds and their geodesics

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Abstract. We describe the geodesics of two-step nilpotent Lie groups N with respect to left invariant Riemannian metrics $\langle.,.\rangle$ using the Riemannian submersion structure of the fiber bundle $\pi : N \to N/\mathcal{Z}$, where $\mathcal{Z}$ denotes the center of N. We characterize two-step nilmanifolds $(N, \langle.,.\rangle)$ which have the property that the projections of geodesics of N onto the Euclidean factor space $N/\mathcal{Z}$ are planar curves.

On certain Ricci-pseudosymmetric hypersurfaces in space forms

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Curvature properties of hypersurfaces in semi-Riemannian space forms are presented.
First we discuss quasi-Einstein hypersurfaces satisfying certain weak curvature condition of pseudosymmetry type. We prove that such hypersurface must be Ricci-pseudosymmetric and we obtain some relation between scalar curvatures of hypersurface and ambient space.

Next we consider pseudosymmetric hypersurfaces and we find the necessary and sufficient conditions for a such hypersurface to be quasi-Einsteinian. We also give an example of a nonpseudosymmetric Ricci-pseudosymmetric quasi-Einstein hypersurface.

Finally, we consider curvature properties of hypersurfaces satisfying some generalized Einstein metric condition. Our main result states that such hypersurfaces must be Ricci-pseudosymmetric.

Second neighbourhood of the diagonal, and a foundation for conformal geometry

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Abstract

A pseudo-riemannian metric g on a manifold M may be described as a function $g : M_{(2)} \to R$, vanishing on $M_{(1)}$, and with a certain non-degeneracy property, cf. [2]. Here, $M_{(k)}$ denotes the "k'th neighbourhood of the diagonal", the scheme represented by the ring of functions $M \times M \to R$ modulo the ideal of those that vanish to the $k+1$st order on the diagonal $M \subseteq M \times M$.

A pseudo-riemannian metric gives rise to some combinatorial structure on the manifold, the Levi-Civita *connection* (parallelism), the associated geodesic *spray*, and the associated *partial exponential map*. This is classical; in the synthetic way of speaking, these data get a formulation which is at the same time very economical and very geometric (notably the parallelism).

Thus, the partial exponential map may be construed as a bijection between $M_{(2)}$ and the (fibrewise) "second neighbourhood $D_{(2)}(TM)$ of the zero section in $T(M)$" where $T(M)$ is the tangent bundle of M. Thus

$$\exp : D_{(2)}(TM) \xrightarrow{\cong} M_{(2)};$$

the inverse of this map we of course denote $\log : M_{(2)} \to D_{(2)}(TM)$. The data of such exp is *equivalent* to a spray or (torsion free) connection, cf. [1].

(The restriction of log and exp to the *first* order neigbourhood of the diagonal, respectively, the first order neigbourhood of the zero section, however, contains *no* data, since they are uniquely determined on *any* smooth manifold M by virtue of the fact that affine combinations of 1-neighbours is well defined on any manifold, and preserved by any smooth map cf. [4] §2.)

The metric g gives rise to an inner product $< -, - >$ on each vector space $T_x(M)$ ($x \in M$); the recipe for this does not depend on exp and log; but using exp, we can define a further structure on M, which has not been considered classically, namely what we call the *Laplacian neigbourhood of the diagonal*, M_L (cf. [3]); it is contained in the second neighbourhood of the diagonal, in fact

$$M_{(1)} \subseteq M_L \subseteq M_{(2)}.$$

It suffices to describe a subset $D_L(TM) \subseteq D_{(2)}(TM)$, and then "transport it back to M using exp. The description of $D_L(TM)$ is made fibrewise, so we describe

$D_L(T_xM) \subseteq D_{(2)}(T_xM)$. This only depends on T_xM being a finite dimensional vector space with an inner product

$$< -, - >: T_xM \times T_xM \to R.$$

Definition Let V be an n-dimensional vector space with an inner product $< -, - >$. A vector $a \in V$ is called *L-small* if for all $u, v \in V$

$$< a, u > < a, v > = \frac{1}{n} < a, a > < u, v > \tag{1}$$

The set of L-small vectors is denoted $D_L(V)$. For R^n with standard inner product, we describe $D_L(R^n)$ in coordinates: a vector $t = (t_1, \ldots, t_n) \in R^n$ is L-small precisely when

$$t_1^2 = \ldots = t_n^2; \text{ and } t_i t_j = 0 \text{ for } i \neq j.$$

This $D_L(R^n)$ is the geometric object described by a certain Weil algebra of linear dimension $n+2$, a quotient of the Weil algebra describing $D_2(R^n)$ (the "2-jet classifier in dimension n") and which is of linear dimension $1 + n + n(n+1)/2$).

Henceforth, we assume that the metric g is positive definite, in the sense that $< -, - >$ is positive definite. In this case, we think of $g(x, y)$ as the *square distance* between x and y, where $(x, y) \in M_{(2)}$; (g is automatically symmetric in its arguments).

Two metrics g_1 and g_2 are conformally equivalent if there exists a function f with positive values so that

$$g_2(x, y) = f(x) g_1(x, y),$$

for all $(x, y) \in M_{(2)}$. Conformally equivalent metrics give rise to the *same* parallelism/spray/partial exponential map, and to *proportional* inner products on the tangent spaces. Clearly, if two inner products on a vector space are proportional, $< -, - >_2 = k < -, - >_1$, they define the same L-small vectors. We conclude: conformally equivalent metrics on M define the *same* $M_L \subseteq M_{(2)}$.

In [3], we proved that a diffeomorphism $h : N \to M$ between Riemannian manifolds is conformal if and only if it preserves L-neighborhoods,

$$(x, y) \in N_L \text{ implies } (h(x), h(y)) \in M_L.$$

But this latter property of course makes sense independently of whether h is a diffeomorphism, and independently of whether or not M and N have the same dimension. However, the condition implies, under mild assumptions, that h is a submersion. In fact, we shall characterize maps preserving the L-neigbourhoods in terms of linear-algebra properties of their differential.

Isometries h have the further property that $g(x, y) = g(h(x), h(y))$, whenever $(x, y) \in N_L$. This also generalizes: if the dimensions of N and M are n and m, respectively, we may consider maps $h : N \to M$ which preserve the L-neigbourhoods and which furthermore satisfy

$$\frac{1}{n} g(x, y) = \frac{1}{m} g(h(x), h(y)),$$

for all $(x, y) \in N_L$. We also characterize such maps in linear-algebra terms.

As an aspect of the parallelism/partial-exponential map structure on a Riemannian manifold, we have a structure allowing us to form *affine combinations* of any pair $(x, y) \in M_{(2)}$, e.g. their midpoint, or the mirror image of y in x. The set $M_L \subseteq M_{(2)}$ is stable under such constructs. So for a map preserving L-neighbourhoods, we may furthermore ask that it preserves such affine combinations. This turns out to be a notion of *harmonic* map.

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Prolongation of projectable tangent valued forms

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We study the prolongation of projectable tangent valued k-forms on fibered manifolds with respect to a bundle functor of rather general type that is based on the flow prolongation of vector fields and uses an auxiliary linear r-order connection on the tangent bundle of the base manifold.

Remarks on Fibonomial Calculus

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As every one knows Fibonacci numbers(1202) [7, 8] form a sequence of integrals satysfying the reccurence formula:

$$F_{n+2} = F_{n+1} + F_n, \quad F_1 = F_2 = 1.$$

This sequence even today is the subject of continuing research, especially by the Fibonacci Association which publishes "The Fibonacci Quaterly". Fibonacci sequence has a lot of interesting properties [5, 8], for example: some divisibility properties and completeness with respect to $\mathbf{N}$.
This is an indicatory presentation of some definitions and theorems of Fibonomial Calculus which is a special case of ψ-extented Rotaś finite operator calculus [1].

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Symplectic structures of the tangent bundles of symplectic and cosymplectic manifolds
(short version)

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Abstract: We describe all symplectic structures on the tangent bundles of symplectic and cosymplectic manifolds.

0. Introduction

In [4], the authors classified all Hamiltonian type natural operators on the cotangent bundle T^*M. In [1], the author described all Hamiltonian type natural operators transforming a function f on a symplectic manifold (M, ω) into a vector field $V(\omega, f)$ on M. The present note deals with canonical constructions on symplectic structures, too. We describe all symplectic structures on the tangent bundles of symplectic and cosymplectic manifolds. This problem arises in the context of respective natural operators in the sense of [5], which are defined on symplectic (resp. cosymplectic) structures. Since homotheties are not symplectomorphisms (resp. cosymplectomorphisms), it is difficult to apply the homogeneous function theorem and the problem of classifications of the operators in question is more difficult than the one of natural operators defined on all 2–forms, [2], [3], [6], e.t.c.

Symplectic structures are involved in the equation of motion. That is why, the results of the paper are interesting with respect to the theoretical mechanics. They are also interesting with respect to the theory of natural operators.

We start with the problem how to construct canonically a symplectic manifold $(TM, \Lambda(\omega))$ for a given symplectic $2m$–manifold (M, ω), where ω is a closed 2–form with $\omega^m \neq 0$ for any point in M. This problem arises in the context of respective $\mathcal{M}f_{2m}$– natural operators Λ. The first main result of the present note is the following classification theorem.

Theorem 1. *Let Λ be a $\mathcal{M}f_{2m}$ –natural operator in question. Then there exist real numbers α and $\beta \neq 0$ such that*

$$\Lambda(\omega) = \alpha\pi^*\omega + \beta\tilde{\omega}^*\Omega \tag{1}$$

for any symplectic structure ω on M, where $\pi^*\omega$ is the vertical lifting of ω to the tangent bundle TM, $\pi : TM \to M$ is the tangent bundle projection, $(\ )^*$ is the pull–back, Ω is the well–known canonical symplectic structure on the cotangent bundle

Key words: natural operators, symplectic structures, cosymplectic structures
AMS Classification: 58 A 20

T^*M and $\tilde{\omega} : TM \to T^*M$ is the standard isomorphism induced by ω, $\tilde{\omega}(v) = \omega_x(., v)$, $v \in T_xM$, $x \in M$.

On the other hand for any real numbers α and $\beta \neq 0$ the operator $\Lambda(\omega)$ defined as in (1) *is a symplectic structure on TM.*

Theorem 1 is a consequence of the following more general fact.

Theorem 2. *Let Λ be a $\mathcal{M}f_{2m}$-natural operator transforming a symplectic structure ω on a $2m$-manifold M into* 2*-form $\Lambda(\omega)$ on TM. Then there exist real numbers α and β such that*

$$\Lambda(\omega) = \alpha\pi^*\omega + \beta\tilde{\omega}^*\Omega \tag{2}$$

for any symplectic structure ω on M, where Ω and $\tilde{\omega}$ are as in Theorem 1.

Using the isomorphism $\tilde{\omega} : TM \to T^*M$ induced by ω we can obtain respective versions of Theorems 1 and 2 for T^*M instead of TM.

Next, we study the problem how to construct canonically a symplectic manifold $(TM, \Lambda(\omega, \theta))$ for a given cosymplectic $2m+1$-manifold (M, ω, θ), where ω is a closed 2-form and θ is a closed 1-form with $\omega^m \wedge \theta \neq 0$ for any point in M. This problem arises in the context of respective $\mathcal{M}f_{2m+1}-$natural operators Λ. The second main result of the present note is the following classification theorem.

Theorem 3. *Let Λ be a $\mathcal{M}f_{2m+1}$-natural operator in question. Then there exist an uniqely determined real number a and uniquely determined smooth maps $b, c : \mathbf{R} \to \mathbf{R}$ with $b(x) \neq 0$ and $b(x) + c(x) \neq 0$ for all $x \in M$ such that*

$$\Lambda(\omega, \theta) = a\pi^*\omega + (b \circ \theta)\varphi^*_{\omega,\theta}\Omega + (b' \circ \theta)\varphi^*_{\omega,\theta}\lambda \wedge d\theta + (c \circ \theta)\pi^*\theta \wedge d\theta \tag{3}$$

for any cosymplectic structure (ω, θ) on M, where λ is the standard Liouville 1*-form on the cotangent bundle T^*M, $\Omega = -d\lambda$ is the well-known canonical symplectic structure on T^*M, $\pi : TM \to M$ is the tangent bundle projection, $\varphi_{\omega,\theta} : TM \to T^*M$ is the standard isomorphism induced by (ω, θ), $\varphi_{\omega,\theta}(v) = \omega_x(., v) + \theta(v)\theta_x$, $v \in T_xM$, $x \in M$, $(\ )^*$ is the pull-back and $d\theta$ is the differential of $\theta : TM \to \mathbf{R}$.*

On the other hand for any real number a and smooth maps $b, c : \mathbf{R} \to \mathbf{R}$ with $b(x) + c(x) \neq 0$ and $b(x) \neq 0$ for all $x \in M$ the operator $\Lambda(\omega, \theta)$ defined as in (3) *is a symplectic structure on TM.*

Theorem 2 is a consequence of the following more general Theorem 4.

Theorem 4. *Let Λ be a $\mathcal{M}f_{2m+1}$-natural operator transforming a cosymplectic structure (ω, θ) on a $2m+1$-manifold M into* 2*-form $\Lambda(\omega, \theta)$ on TM. Then there exist uniquely determined smooth maps $\alpha, \beta, \gamma, \delta, \epsilon : \mathbf{R} \to \mathbf{R}$ such that*

$$\begin{aligned}\Lambda(\omega, \theta) = (\alpha \circ \theta)\varphi^*_{\omega,\theta}\Omega + (\beta \circ \theta)\pi^*\omega + (\gamma \circ \theta)\varphi^*_{\omega,\theta}\lambda \wedge \pi^*\theta + \\ (\delta \circ \theta)\varphi^*_{\omega,\theta}\lambda \wedge d\theta + (\epsilon \circ \theta)\pi^*\theta \wedge d\theta\end{aligned} \tag{4}$$

for any cosymplectic structure (ω, θ) on M, where λ, $\Omega = -d\lambda$, $\varphi_{\omega,\theta}$, π and $(\ )^*$ are as in Theorem 3.

Using the isomorphism $\varphi_{\omega,\theta} : TM \to T^*M$ induced by (ω, θ) we can obtain respective versions of Theorems 3 and 4 for T^*M instead of TM.

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Extended finite operator calculus - an example of algebraization of analysis

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Abstract

"A Calculus of Sequences" started in 1936 by Ward constitutes the general scheme for extensions of classical operator calculus of Rota - Mullin considered by many afterwards and after Ward. Because of the notation we shall call the Ward's calculus of sequences in its afterwards elaborated form - a ψ-calculus.

The ψ-calculus in parts appears to be almost automatic, natural extension of classical operator calculus of Rota - Mullin or equivalently - of umbral calculus of Roman and Rota.

At the same time this calculus is an example of the algebraization of the analysis - here restricted to the algebra of polynomials. Many of the results of ψ-calculus may be extended to Markowsky Q-umbral calculus where Q stands for a generalized difference operator, i.e. the one lowering the degree of any polynomial by one.

This is a review article based on the recent author's relevant contributions. The article is supplemented by the short indicatory glossaries of terms and notation used by Ward, Viskov, Markowsky, Roman on one side and the Rota-oriented notation on the other side.

On the Bochner curvature of para–Käehlerian manifolds

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An $n(=2m)$-dimensional differentiable manifold M is said to be a para-Kählerian manifold if it admits a $(1,1)$-tensor field J and pseudo-Riemannian metric g on M such that

$$J^2 = I, \qquad g(JX, JY) = -g(X, Y), \qquad \nabla J = 0,$$

where ∇ is the Levi-Civita connection of g and I is the identity tensor field. The pair (J, g) is called a para-Kählerian manifold structure on M. Let B be the Bochner curvature tensor of a para-Kählerian manifold (M, J, g). We show that if the manifold is Bochner parallel ($\nabla B = 0$), then it is Bochner flat ($B = 0$) or locally symmetric ($\nabla R = 0$). We also consider the para-Kählerian manifolds whose Ricci curvature is paraholomorphically pseudosymmetric. We find necessary and sufficient conditions for a Bochner flat para-Kählerian manifold to be paraholomorphic Ricci-pseudosymmetric.

Transitive Lie algebroids: spectral sequences and signature

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We prove that for any transitive unimodular invariantly oriented Lie algebroid L on a compact, oriented and connected manifold with isotropy Lie algebra $\mathfrak{g}$ and trivial monodromy the cohomology algebra is the Poincaré algebra with trivial signature. In particular, the examples of such algebroids are when M is simply connected or when $\operatorname{Aut} G = \operatorname{Int} G$, for simply connected Lie group G with the Lie algebra $\mathfrak{g}$, or when the adjoint Lie algebra bundle $\boldsymbol{g}$ induces trivial homology bundle $H^*(\boldsymbol{g})$ in the category of flat bundles.

Geometric singularity classes for special k-flags ($k \geqslant 2$)

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1 Definition of special k-flags and the aim

Special k-flags ($k \geqslant 2$) of all lengths $r \geqslant 1$ have been defined in [M] by conditions *formally* stronger than the conditions defining in [PR] 'generalized contact systems for curves', or else than those of [KRub] putting in evidence 'k-flags satisfying certain normality conditions'. The reason was that precisely such conditions were prompted by one structural theorem in [BH][1] that, in [M], was generalized by means of *multi-dimensional Cartan prolongations*.

Under closer inspection, using two early Bryant's results quoted in [PR] as well as one original lemma of the authors of [PR], the two definitions (or three, taking into account also that of [KRub]) boil down to the same.

Namely, the tower of consecutive Lie squares of D

$$TM = D^0 \supset D^1 \supset D^2 \supset \cdots \supset D^{r-1} \supset D^r = D$$

(that is, $D^{j-1} = [D^j,\, D^j]$ for $j = r,\, r-1, \ldots,\, 2,\, 1$) should consist of distributions of ranks, starting from the smallest object D^r: $k+1,\, 2k+1,\, \ldots,\, rk+1,\, (r+1)k+1 = \dim M$ such that

$\star$ for $j = 1, \ldots, r-1$ the Cauchy-characteristic module $L(D^j)$ of D^j sits already in the smaller object D^{j+1}: $L(D^j) \subset D^{j+1}$ **and** is regular of corank 1 in D^{j+1}, while $L(D^r) = 0$;

$\star\star$ the *covariant subdistribution* F of D^1 (see [KRub], p. 4 for the definition extending the classical Cartan approach of 1910, cf. also [MPel]) is involutive and of corank 1 in D^1 (hence also regular)[2].

Local polynomial pseudo-normal forms, so-called EKR's (Extended Kumpera Ruiz) for such D were started in [KRub] and [PR], then fully constructed only in [M], after

[1] attributed by Bryant & Hsu to E. Cartan

[2] this additional requirement 'corank 1 for F in D^1' is superfluous once that covariant object is assumed to be involutive, cf. Lem. 1 in [KRub]

which a question had appeared about the geometrical meaning and significance of different families of those pseudo-normal forms.

> Just like a similar question for Goursat flags [that is – outside the scope of the present abstract – when $k = 1$; the definition of 1-flags is more compact and simpler] pending several years after [KRui], started to be settled only in [BH], leading eventually, for any fixed length r, to 2^{r-2} Kumpera-Ruiz singularity classes in [MonZ], encoded by the words over $\{*, \mathrm{S}\}$ and exactly corresponding to the original pseudo-normal forms of [KRui].

2 Different words associated to germs of special k-flags, including the word 'singularity class'

We intend to present at Krynica'03 a line of very recent research aimed at finding invariant singularity classes of special k-flags that put on a solid geometrical basis the cornucopia of pseudo-normal forms found by us a year ago and reported in [M].

Within this chapter, we keep the germ of a rank-$(k+1)$ distribution D at $p \in M$, generating on M a special k-flag of length r, fixed. For any fixed $1 \leqslant m \leqslant k$, a word $j_1.j_2 \ldots j_r$ over the alphabet $\{1, 2, \ldots, m-1, \underline{m}\}$ (with the last letter underlined!) is *admissible* when it starts with $j_1 = 1$ and satisfies the rule of the least upward jumps introduced in Thm. 3, [M]: for $l = 1, 2, \ldots, r-1$, if $j_{l+1} > \max(j_1, \ldots, j_l)$ then $j_{l+1} = 1 + \max(j_1, \ldots, j_l)$.

So that, for example for $m = 1$, admissible are all the words $\underline{1}.\underline{1} \ldots \underline{1}$. For $m = 3 \leqslant k$ and $r = 4$ admissible are: 1.1.1.1, 1.1.1.2; 1.1.2.1, 1.1.2.2, 1.1.2.$\underline{3}$; 1.2.1.1, 1.2.1.2, 1.2.1.$\underline{3}$, 1.2.2.1, 1.2.2.2, 1.2.2.$\underline{3}$, 1.2.$\underline{3}$.1, 1.2.$\underline{3}$.2, 1.2.$\underline{3}$.$\underline{3}$.

Suppose that in an admissible word $\mathcal{C}$ there appears somewhere, for the first time when going from the left, the letter $\underline{m} = j_l$ **and** that there are in $\mathcal{C}$ other letters $\underline{m} = j_s$, $l < s$, as well. (In the example above only $\mathcal{C} = 1.2.\underline{3}.\underline{3}$ is of this type.) Suppose also that with any such $\underline{m} = j_s$ there is **associated a module** V of local smooth vector fields on (M, p). In such an abstract situation we will specify that letter $\underline{m}$ to one of the two: to m or else to $\underline{m+1}$. The specification will clearly depend on that module V but, needless to say, also on the germ of D at p (and, in applications in the subsequent section 2.2, V itself will always be some object derived, sometimes remotely, from D).

The definition will be inductive on $m \in \{1, 2, \ldots, k\}$, for admissible words of lengths $\leqslant r$, with the beginning being fairly simple. The step, however, will be involved.

2.1 The key definition.

The beginning of induction for $m = 1$. $\underline{1} = j_s$; $2 \leqslant s \leqslant r$ (the word has length $\leqslant r$, not necessarily equal to r). When $s = 2$, j_s becomes 1 when $F(p) \not\supset V(p)$, and becomes $\underline{2}$ when $F(p) \supset V(p)$.

When $3 \leqslant s \leqslant r$, then j_s becomes 1 when $L(D^{s-2})(p) \not\supset V(p)$, and becomes $\underline{2}$ when $L(D^{s-2})(p) \supset V(p)$.

The step of induction $m \Rightarrow m+1$ for $1 \leqslant m \leqslant k-1$. We assume that for **any** admissible word $\mathcal{C}$ of length $\leqslant r$ over $\{1, 2, \dots, m-1, \underline{m}\}$, and then for **any** module V attached to a non-first letter $\underline{m}$ in $\mathcal{C}$, that module V knows how to specify 'its' letter $\underline{m}$ to either m or $\underline{m+1}$.

Now we take any admissible word $\mathcal{C} = j_1.j_2.j_3\dots$ of length $\leqslant r$ over $\{1, 2, \dots, m, \underline{m+1}\}$ with more than one letter $\underline{m+1}$, and a module V of vector fields on (M, p) attached to a non-first letter $\underline{m+1} = j_t$ in $\mathcal{C}$. We precise also, and this is very important, that the nearest to j_t – always to the left in $\mathcal{C}$ – is the letter $\underline{m+1} = j_s$, $s < t$. These two 'neighbouring' letters $\underline{m+1}$ may possibly be separated in $\mathcal{C}$ by an ocean of letters smaller than $m+1$.

Our aim now is to transport backwards, or rather *transform* V, being attached to the t-th place, *into* another module W attached to the s-th place. In other words, we want to sail with V through that ocean of smaller letters to the nearest to the left harbour $\underline{m+1}$.

Going backwards from j_t towards j_s, one meets firstly $l \geqslant 0$ letters 1, then a letter from $\{2, 3, \dots, m\}$, then again $n \geqslant 0$ letters 1, then a letter from $\{2, 3, \dots, m\}$, and so on until arriving to the harbour j_s. Possibilities are really various: there can be just one l and nothing more (when there occur only letters 1 between j_s and j_t) and that l can even vanish, as it happens in the example $1.2.\underline{3}.\underline{3}$ with $j_3 = \underline{3} = j_4$, or else there can be several $l = n = \dots = 0$ (think about $1.2.\underline{3}.2.\underline{3}$), or else…

The gist of the construction consists in taking the **small** flag of V, built out of modules of vector fields on M,

$$V = V_1 \subset V_2 \subset V_3 \subset V_4 \subset V_5 \subset \cdots,$$

$V_{i+1} = V_i + [V_1, V_i]$, then starting another small flag, departing precisely from the member V_{3+2l} of the previous one,

$$V_{3+2l} = U_1 \subset U_2 \subset U_3 \subset U_4 \subset \cdots,$$

$U_{i+1} = U_i + [U_1, U_i]$, then possibly starting yet another small flag departing that time from the member U_{3+2n}, and so on possibly many times. The number of small flags involved is equal to the number of letters bigger than 1 (and, naturally, smaller than $m+1$) in between j_s and j_t. If there occurs only one such letter, and hence only the intermediate values l and n are defined, then the sailing terminates by $U_{3+2n} = W$. If there are, say, ten letters exceeding 1 in between j_t and j_s, and the number of 1's in row directly before arriving at j_s is $N \geqslant 0$, then eleven small flags are used in the transport – or transfert – and precisely the $(3+2N)$-th member of the eleventh small flag is, by our definition, the module W.

If there is a sea of letters bigger than 1 in that ocean of letters separating j_t from j_s, then it takes ages to transform the input module V into the output module W.

Now the truncated word $\mathcal{C}' = j_1.j_2\dots j_s$ (also admissible because truncation preserves admissibility) will be transformed into an admissible word $\mathcal{E} = i_1.i_2\dots i_s$ over$\{1, 2, \dots, m-1, \underline{m}\}$. Namely, $\mathcal{E}$ is being obtained from $\mathcal{C}'$ by replacing all letters $\underline{m+1}$ (there is at least one such letter in $\mathcal{C}'$: j_s) **and** all letters m (there is at least one such letter before j_s: $\mathcal{C}'$ is admissible) by $\underline{m}$.

Thus $i_s = \underline{m}$ and this $\underline{m}$ is not the first such letter in $\mathcal{E}$. At that, remembering that W has been attached to j_s in $\mathcal{C}'$, we can now treat the module W as attached to i_s in $\mathcal{E}$. In this situation, by the induction hypothesis, W knows how to specify further its letter $\underline{m}$.

• If W specifies its i_s in $\mathcal{E}$ to m then we simply declare that, in the word $\mathcal{C}$, the module V specifies its $j_t = \underline{m+1}$ to $m+1$.

•• If W specifies its i_s in $\mathcal{E}$ to $\underline{m+1}$ then we say that, in $\mathcal{C}$, V specifies its $\underline{m+1}$ to $\underline{m+2}$.

Endly, it is needed to pose for a while on the last round of this long induction procedure. One passes then from the specification(s) of $\underline{m} = \underline{k-1}$ to the specification(s) of $\underline{m+1} = \underline{k}$ (to either k or $\underline{k+1}$). Now the entire definition is being concluded by erasing the underlines in all obtained letters $\underline{k+1}$ (if any). It is because of our dealing precisely with k-flags, when $k+1$ is the last letter in the target alphabet (that letter has not to be specified any further).

SUMMARIZING this section, modules of vector fields on the underlying manifold, attached to letters being underlined in admissible words, know how to refine those letters to more sophisticated ones.

2.2 Definition of the singularity class of a [germ of] special k-flag.

As announced earlier in this abstract, for the germ of D at $p \in M$, D generating on M a special k-flag of length r, we will define its **singularity class**, thus generalizing Kumpera-Ruiz singularity classes of [germs of] 1-flags. This cardinal geometric object will be given under the form of a certain admissible word $\mathcal{W}(D)$ of length r over $\{1,\, 2, \dots,\, k,\, k+1\}$. The construction of $\mathcal{W}(D)$ will be stepwise, and in the meantime we will pass by all intermediate alphabets $\{1,\, 2, \dots,\, m-1,\, \underline{m}\}$ for $m = 1,\, 2, \dots,\, k$.

To begin with, we take the most primitive word $\mathcal{C}_1 = \underline{1}.\underline{1}\dots\underline{1}$ of length r and specify, independently, to either 1 or $\underline{2}$ all its letters except the first $\underline{1}$, by applying our key definition of Sec. 2.1 for $m = 1$.
Yet, to this end, one has to have modules of vector fields attached to these letters! And so (no wonder perhaps) to the j-th letter, $j = 2,\, 3, \dots,\, r$, we just attach the member D^j [i. e., a *regular* module of vector fields on M] of the flag of $D = D^r$. Then we replace the first letter $\underline{1}$ in $\mathcal{C}_1$ by 1. The outcome is an admissible word $\mathcal{C}_2$ of length r over $\{1,\, \underline{2}\}$. When it contains no letters $\underline{2}$ then the construction ends and $\mathcal{W}(D) = \mathcal{C}_2$. When it does contain any letter $\underline{2}$, we pass to the next step.

At the next step of our *computing* the word $\mathcal{W}(D)$, we work with $\mathcal{C}_2$. If it contains more than one letter $\underline{2}$ then we apply, this time for $m = 2$, the machinery of Sec. 2.1 to all non-first letters $\underline{2}$ (independently to any one such letter) in $\mathcal{C}_2$. And with what modules of vector fields attached to those letters? As previously, nothing but the respective members D^j of the flag under consideration. And then we replace the first $\underline{2}$ in $\mathcal{C}_2$ by 2.
If $\mathcal{C}_2$ contains just one letter $\underline{2}$ then we replace it by 2.

All in all, we arrive at the word $\mathcal{C}_3$, of inchanged length r, that by construction is admissible over $\{1,\, 2,\, \underline{3}\}$. When there is no $\underline{3}$ in $\mathcal{C}_3$ then $\mathcal{W}(D) = \mathcal{C}_3$ and ... the geometry of D in the vicinity of p does not 'trouble' us any more. In the opposite case we pass to the next step, producing the word $\mathcal{C}_4$.

And so on, this line of computation may either end at some step, or else it may last until the very last phase. For instance (but not only then) it ends 'prematurely' when the length r is not big enough in comparison to k (when $r \leqslant k$). When it lasts till the end, how does the last phase look like?

That last phase is necessary when the geometry of D around p has appeared so rich (including, clearly, the length being sufficiently big) as to have forced us to produce the admissible word $\mathcal{C}_k$ over $\{1,\, 2, \dots,\, k-1,\, \underline{k}\}$ that does feature letter(s) $\underline{k}$.
When there is just one such letter, in the final phase it is automatically replaced by k. When there are several letters $\underline{k}$ in $\mathcal{C}_k$ then to all non-first-from-the-left such letters we apply the mechanism of Sec. 2.1 for $m = k$, always with the respective module D^j attached to the letter $\underline{k}$ [under consideration] that appears as the j-th letter in $\mathcal{C}_k$. The geometry of the flag, in disguise of the proposed involved algorithm, decides then whether such $\underline{k}$ is to be specified to k or else to the biggest letter $k+1$ (with no line underneath!, cf. the end of Sec. 2.1).

3 Main result

To no surprise, the singularity classes surge to surface in those mentioned local polynomial pseudo-normal forms EKR for special flags, obtained in [M]. Or else, watching the same phenomenon from another angle when *only* an EKR is available, its singularity class is **clearly** visible in its very construction. Saying still otherwise, the EKR's are faithful to the underlying local flag's geometries. That is to say, there holds

Theorem [2003]. For every germ D of a rank-$(k{+}1)$ distribution generating a special k-flag of length $r \geqslant 1$, and for *every* its pseudo-normal form $j_1 .\, j_2 \dots j_r$ issuing from [M], the word $j_1 .\, j_2 \dots j_r$ is but $\mathcal{W}(D)$.
Also conversely, each germ E already being in a pseudo-normal form $j_1 .\, j_2 \dots j_r$ [that form subject to the least upward jumps rule of [M], and with constants – wherever allowed by that shell form – arbitrarily fixed] has its singularity class $\mathcal{W}(E) = j_1 .\, j_2 \dots j_r$.

A proof of this theorem will be given in a subsequent paper.

3.1 How many singularity classes exist and of what codimensions they are.

These two auxiliary questions clearly impose by themselves.

On each manifold M of dimension $(r+1)k+1$ bearing a special k-flag of length r, the shadows of singularity classes (one says also about *materializations* of singularities) form always – and not only for 'generic' flags! – a very neat stratification by embedded submanifolds whose codimensions are directly computable.

Namely, the codimension of the materialization of any fixed singularity class $\mathcal{C}$ is equal, if only the materialization is not empty, to

the number of letters 2 in $\mathcal{C}$ + twice the number of letters 3 in $\mathcal{C}$
+ thrice the number of letters 4 in $\mathcal{C} + \cdots$
+ k times the number of letters $k+1$ in $\mathcal{C}$.

Once Theorem shown, one proves this statement locally, using any fixed EKR depicting locally the flag in question.

As to the numbers of different singularity classes, one computes them recursively with respect to k, keeping r fixed. These computations are not so straightforward as the preceding ones for codimensions, although can still be kept under control (and an algorithm for them can be written). The starting point is, naturally, the situation $k = 2,\ r \geqslant 3$: the number of such singularity classes equals $2 + 3 + 3^2 + \cdots + 3^{r-2}$. For instance, for $r = 7$ there are 365 such classes. To offer a glimpse of the growth, this can be compared with the number 715 for the same $r = 7$ but with this time $k = 3$ instead of 2.

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Invariant Hyper-Kähler Structures on the Cotangent Bundles of Hermitian Symmetric Spaces

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Let $M = G/K$ be an irreducible Hermitian symmetric space of compact type with a homogeneous metric $\mathbf{g}_{\mathrm{M}}$. Since M is a homogeneous complex manifold, its cotangent bundle T^*M has a natural complex structure. Using $\mathbf{g}_{\mathrm{M}}$ we can identify the cotangent and tangent bundles and thus obtain a complex structure on TM, with respect to which the zero section $M \subset TM$ is complex. This structure J^- is different from the standard complex structure J^+ on TM induced by that on M.

On the other hand, the cotangent bundle $T^*M \simeq TM$ is a symplectic manifold with the canonical symplectic form Ω. We make an explicit description of all G-invariant Kähler structures (J, Ω) (with the Kähler form Ω) on homogeneous domains $D \subset TM$ anticommuting with J^-. In fact, each resulting hypercomplex structure, together with the suitable metric $\mathbf{g}$, defines a hyper-Kählerian structure.

If the domain D contains the zero section M, the restriction of the hyper-Kählerian metric $\mathbf{g}$ to M is the given homogeneous metric $\mathbf{g}_{\mathrm{M}}$ up to a constant multiplier (one makes this multiplier $= 1$ using for the identification of T^*M and TM a homogeneous metric on M proportional to $\mathbf{g}_{\mathrm{M}}$). Such hyper-Kählerian metrics have been constructed in [Bu] using twistor methods and case by case the classification of symmetric spaces, in [Bi] using Nahmś equations and in [DSz] (for spaces of classical groups) using deformation of the so-called adapted complex structure on TM. In [BG1] Biquard and Gauduchon found explicit formulas for these hyper-Kählerian metrics in terms of some operator-functions $P : \mathfrak{m} \to \mathrm{End}(\mathfrak{m})$ on the space $\mathfrak{m} \simeq T_o(G/K)$, where $o = \{K\}$. These hyper-Kählerian structures are global ones. Our additional structures are not defined on the zero section M. So we cannot talk about a restriction of the corresponding hyper-Kählerian metric to M as in [BG1]. Nevertheless, our expressions for P and potential functions generalize the corresponding formulas of [BG1, BG2].

For proofs in [DSz, BG1, BG2] they used the decomposition of $T(TM)$ between horizontal and vertical directions, induced by the Levi-Civita connection of M. Our approch is based on the fact that $T(G/K)$ is a reduced manifold for the (right) Hamiltonian action of K on TG. We can substantially simplify matters by working as in [My] in the trivial vector bundle $G \times \mathfrak{m}$ which is a level surface for the corresponding moment map. So we use the natural homogeneous decomposition of $T(G \times \mathfrak{m})$ usual for the Lie algebras theory. As an application we obtain a new simple proof of the well-known Harish-Chandra and Moore theorem about restricted root systems of Hermitian symmetric spaces.

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The classification of compact smooth Bol loops

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Let be given a connected Lie group G equipped with its natural invariant affine symmetric space structure. We say that G has a totally geodesic decomposition if there exists a proper subgroup $H \subset G$ and a totally geodesic submanifold $M \subset G$ through the unit element $e \in G$ such that the submanifold $M \subset G$ is not a subgroup of G and the set M is a full representative system of the cosets $gH \in G/H,\ g \in G$. Then a differentiable section $\sigma : G/H \to G$ can be defined by $\sigma(gH) = gH \cap M$. If the subgroup H contains no nontrivial normal subgroup of G and M generates G then the totally geodesic decomposition of the Lie group G gives a necessary ad sufficient condition for the exintence of a smooth Bol loop such that the group generated by the left translation would be isomorphic to the given group G. We give a classification of totally geodesic decompositions for compact Lie groups.

The apply the results to the classification of compact connected differentiable Bol loops.

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Quadratization of Lie algebroids

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Abstract. Let Π be a smooth Poisson tensor on $\mathbb{R}^n$ such that $\Pi(0) = 0$. Then, the linear part of the Taylor expansion at 0 of Π defnes a Lie algebra structure on the vector space of linear functions on $\mathbb{R}^n$. According to Alan WEINSTEIN, a Lie algebra L is formally (resp. smoothly or analytically) nondegenerate if every Poisson tensor on $\mathbb{R}^n$, say Π, which vanishes at 0 and whose linear part at 0 is isomorphic to L is formally (resp. smoothly or analytically) linearizable at 0. By a brute force, Jean-Paul DUFOUR and Nguen Tien ZUNG recently prove that the algebra $\mathrm{aff}(n)$ of affine endomorphisms of $\mathbb{R}^n$ is non analytically denegenerate. The aim if this talk is to sketch the proof of the formal nondegenerancy $\mathrm{aff}(n)$ and to extend that nondegenerancy property the class of "AffineLike Lie algebrAs".

On almost complex structures with Norden metrics on tangent bundles

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Abstract. Using the notions of horizontal and vertical lifts, a class of almost complex structures with Norden metrics $(\widetilde{J}, \widetilde{g})$ is defined on the tangent bundle TM of an almost complex manifold with Norden metric $M(J, g)$. Certain sufficient conditions for such a structure to be integrable (complex) are found. Under this conditions, the structure is checked to be a Kähler manifold with Norden metric.

Essential tori in link complements in standard positions: geometric and combinatorial aspects

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Let L be a non-split link represented as a closed braid in S^3 with the braid axis A and let T be an essential(i.e. incompressible and boundary non-parallel) torus in $S^3 \setminus L$ arising in Jaco-Shalen-Johanson decomposition of $S^3 \setminus L$ [3, 4]. Consider the fibration of $S^3 \setminus A$, with the fibers being open discs $\{H_\theta : \theta \in [0, 2\pi]\}$. Then a torus T in the closed braid complement $S^3 \setminus L$ is foliated by its intersections with the fibers H_θ and admits a cell decomposition induced by this foliation. Birman and Menasco showed [2] that any essential torus T in the complement of non-split link may be performed (via isotopies relatively the axis, and the so-called exchange moves) to the one in the three standard types of embeddings. The notion of standard embedding of a torus with respect to the axis is defined in the terms of the natural foliation on T or the cell-decomposition of T induced by this foliation.

In the first case (type 0 embedding), T is transverse to every fiber H_θ and intersects each fiber in a meridian of solid torus V bounded by T. The complete geometric description of type 0 tori can be easily understood from their definition. In the second case (type 1 embedding), the torus T admits the canonical mixed decomposition (foliation). The complete geometric description of type 1 embedded tori has been given by Birman and Menasco [2].

In the third case, the embedded torus T admits a standard tiling [2] . In the present talk, we propose a geometric description of essential tori in closed braid complements which admit standard tiling. For this, we use the combinatorial patterns of standard tilings of these essential tori. In opposite to the tori of types 0 and 1, the class of embedded tori admitting standard tiling have much more complicated geometrical description. In [2], Birman and Menasco gave for each $k \geqslant 2$ a geometric description of the tori of type $k \geqslant 2$, a class of embedded tori which admit standard tiling. The fundamental domain of a type $k \geqslant 2$ torus T is a rectangle of width 2 and height k. Each torus T of type $k \geqslant 2$ has the following geometric description. T is made of k cylinders which are glued together in a cycle. The core of the ith cylinder is an arc α_i which lies entirely in a fiber H and has its endpoints on A. The core of the solid torus V bounded by T, in general, is knotted. Moreover each cylinder intersects A twice so that $|T \cap A| = 2k$. However the class of tori of types $\geqslant 2$ does not exhaust all embedded tori which admit standard tiling. In [6], Ng described all possible standard tilings of essential tori in link complements via the so-called staircase tiling patterns and showed that every embedded torus T which admits a standard tiling possesses a staircase tiling pattern of even width $2n$ and height $k \geqslant 2$. Any such a torus T can be obtained from a staircase pattern by identifying two opposite zig-zag sides with a possible shift l. Therefore any staircase pattern is actually characterized by a triple of parameters $(2n, k, l)$. Note that each essential torus in a standard position possesses two different staircase tiling patterns, dual of each other. In the present talk, we discuss the problem of geometric realization of $(2n, k, l)$-staircase patterns and the relationship between the parameters of staircase patterns, dual of each other.

Ng [6] described a series of examples of staircase tiling patterns of width $2n$ and height k which admit geometric realization as embedded essential tori in some link complements. All such tori and their patterns are obtained from the standard tori of type $k \geqslant 2$ in the sense of Birman and Menasco, having the 2 by k staircase patterns, by using the operation of "making tracks. She posed the following

Problem 1. Find a complete set of well-defined moves on the embedded tori of type $k \geqslant 2$ such that any embedding of torus which admits a standard tiling can be obtained from one of type $n \geqslant 2$ by a sequence of these moves.

In the present talk, we describe a complete set of well-defined moves on tori such that any embedded torus which admits a standard tiling can be obtained from the one of type $k \geqslant 2$ by applying these moves to it.

As an application of our treatment of embedded essential tori in link complements and their combinatorial patterns, we revisit the methods and techniques, used by J.Los in his dynamical classification of knots. Recall that in [5], Los has suggested a new classification of knots via the dynamical type of their minimal braid representatives. For a given oriented knot K in S^3, let $Bi(K)$ denote the set of braids with the minimal number of strands (called the braid set of K). Any braid $\beta \in B_n$ defines a unique class of homeomorphisms $[f_\beta]$ on the n-punctured disc [2]. By the Nielsen-Thurston theorem [7], isotopy classes of surface homomorphisms are classified into three dynamical types: *periodic, pseudo-Anosov* and *reducible*. Los showed [5] that the Nielsen-Thurston theorem allows to classify the knots by the dynamical type of their minimal braid representatives. The geometric description of essential tori in link complements in standard position plays here important role.

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Lie algebras of differential operators

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The classical result of Pursell and Shanks [PS], which states that the Lie algebra of smooth vector fields of a smooth manifold characterizes the smooth structure of the variety, is the starting point of a multitude of works.

There are similar results in particular geometric situations—for instance for hamiltonian, contact or group invariant vector fields—for which specific tools have each time been constructed, [O, 1, AG, HM], in the case of Lie algebras of vector fields that are modules over the corresponding rings of functions, [Am, G1, S], as well as for the Lie algebra of (not leaf but) foliation preserving vector fields, [G2].

First objective of the lecture is to prove that the Lie algebra $\mathcal{D}(M)$ of all linear differential operators $D : C^\infty(M) \to C^\infty(M)$ of a smooth manifold M determines the smooth structure of M. Beyond this conclusion, it will be presented a description of all automorphisms of the Lie algebra $\mathcal{D}(M)$ (and even of the Lie subalgebra $\mathcal{D}^1(M)$ of all linear first-order differential operators of M) and of the Poisson algebra $S(M) = Pol(T^*M)$ of polynomial functions on the cotangent bundle T^*M (the symbols of the operators in $\mathcal{D}(M)$), the automorphisms of the two last algebras being of course canonically related with those of $\mathcal{D}(M)$. In each situation one obtains an explicit formula. For instance—in the case of $\mathcal{D}(M)$—in terms of the automorphism of $\mathcal{D}(M)$ implemented by a diffeomorphism of M, the conjugation-automorphism of $\mathcal{D}(M)$, and the automorphism of $\mathcal{D}(M)$ generated by the derivation of $\mathcal{D}(M)$ associated to a closed 1-form of M.

The presented results have been obtained in joint work with Janusz Grabowski.

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Lie groupoids viewed as generalized atlases for spaces of leaves

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A smooth manifold structure on a set Q may classically be defined in a twofold way, either considering the (commutative) algebra of smooth functions (from Q to $\mathbb{R}$ or $\mathbb{C}$), or defining an atlas, more precisely the class of its equivalent atlases.

Using a simplified (and perhaps simplistic) language, the first approach will be referred to as the algebraic one, while the secund one will be called geometric.

Now, as is well known, the space of leaves Q of a non–simple (regular) foliation on a smooth manifold is no longer a manifold, though it should seemingly keep some track of smoothness inherited from the ambient manifold, in spite of its very intricate or very poor (possibly coarse) topology.

In that context a generalization of the algebraic approach has been extensively developed by A. Connes, who associated to Q a certain (non–commutative) algebra, or more precisely its Morita equivalence class, and considered Q as one of the basic examples of an object in the so–called Non–Commutative Geometry.

In the present lecture we want to emphasize a (probably less well known and less familiar) generalization of the alternative geometric approach, in which the classical atlases for a manifold Q are described by means of (a very special kind of) smooth groupoids, and the equivalence between various atlases of Q as a (very special occurrence of) Morita equivalence.

More precisely such an atlas may be described by means of an étale surjective map with range Q whose domain is the trivial manifold sum of the codomains of the charts ; the associated (étale) groupoid is then just the graph of the equivalence relation defined by this surjective map, and may be also identified with the pseudogroup of changes of charts. A refinement of an atlas is then described by means of some surjective étale map, along which the associated groupoid has to be pulled back, which gives rise to an equivalent one.

It is then natural to view a general smooth (or Lie) groupoid as a (generalized non–étale) atlas for its orbit space Q, and its Morita equivalence class as defining a (generalized) smooth "structure" on Q. It should be noted that this is not a classical structure on the set Q in the sense of Bourbaki. In such a "structure" any point of Q bares a "structure" given by the isomorphic class of the isotropy group.

This Morita equivalence, which is generally considered in the context of topological groupoids (or sometimes in the more general context of topos theory), will be here defined within the category Dif of smooth manifolds and smooth maps in a very simple way, using diagrams in Dif . Our general policy will be to describe various set–theoretic constructions by means of suitable diagrams of arrows in Set, emphasizing surjections and injections, and then to replace these arrows respectively by surmersions and embeddings in order to transfer these constructions into Dif.

This procedure allows a very simple description of a calculus of fractions for the category of Lie groupoids and smooth morphisms (alias smooth functors), which transforms Morita equivalences into actual isomorphisms and isomorphic functors into the same arrow (which generalizes an outer morphism between groups). The arrows of this category of fractions may be identified (in the topological context) with the morphisms considered by Haefliger and Skandalis–Hilsum. This category seems a very natural framework for Differential Geometry. For example, in that category of fractions, the fundamental group may be characterized by a very simple universal property.

Some general remarks on foliated structures

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Abstract. J. Pradines in [5] introduced and R. Brown and O. Mucuk in [1] developed the notion of a local smooth structure of the groupoid of an equivalence relation. On the other hand, J. Kubarski in [2] proposed the notion of a nice structure of such a groupoid, and proved that, under mild assumption, the existence of this structure characterizes the foliation relations among equivalence relations. In [3] it has been shown that the above notions are notonly equivalent but even identical. The axioms of a nice structure are considerably simpler. We observe that the existence of such structures cannot be extended, in general, to the case of singular foliations. This causes a great difference between regular and singular nontransitive geometries as the existence of "local-nice" structures enables us in the former case to construct a Lie group structure on the automorphism groups of some geometric structures [6]. Another remarks concern categories of foliated manifolds. By using an apparatus from P. Michor [4] we introduce a category of abstract foliated manifolds and foliated mappings (which is a more general concept than foliated structures on Fréchet or convenient manifolds), and try to establish some of its properties. It is proved that in finite dimensions the objects of the category coincide with the usual foliated manifolds. Our definition could be also viewed as a definition of foliations on infinite dimensional manifolds.

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Elliptic operators on singular manifolds and K-homology

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In 1970s Atiyah showed that elliptic operators on a smooth closed manifold define cycles in K-theory. The relationship between elliptic theory and K-theory is even more precise: the group $\mathrm{Ell}(M)$ of stable homotopy classes of elliptic pseudodifferential operators on a manifold M is isomorphic to the even K-homology group of the manifold:

$$\mathrm{Ell}(M) \simeq K_0(M). \tag{1}$$

It turns out that a similar isomorphism holds in many situations, when the manifold is no longer smooth.

1. Manifolds with isolated singularities. Let M be a compact manifold with a finite number of isolated conical points. Denote by $\mathrm{Ell}(M)$ the group of stable homotopy classes of elliptic operators of order zero on M (see, e.g. [1]).

Theorem 1. *On a manifold with isolated conical singularities isomorphism* (1) *holds.*

2. Manifolds with edges of codimension one. Let $\widetilde{M}$ be a compact manifold with boundary and the boundary is represented as the total space of a covering $\pi : \partial\widetilde{M} \to X$ over some base X. The quotient space of the equivalence relation identifying the points in the fibers of the covering is called a manifold with edge X. Denote the quotient by M. Corresponding to the covering on the boundary there is a class of nonlocal operators on $\widetilde{M}$ generated by the usual pseudodifferential operators of order zero on M and operators induced by transpositions of leaves of the covering in a neighborhood of the boundary. We assume that the operators have symbols independent of covariables near the boundary. Denote by $\mathrm{Ell}(M)$ the group of stable homotopy classes of elliptic operators from the class just described.

Theorem 2. *On a manifold with codimension one edges the isomorphism* (1) *holds.*

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The proof of these results is based on Atiyahś functional-analytic description of K-homology as the group of stable homotopy classes of abstract elliptic operators and a generalization of the Atiyah–Singer difference construction to noncommutative algebras of symbols. We describe the former result for the case of conical points.

3. Difference construction on manifolds with conical points. Suppose that the manifold with conical points is obtained from a compact manifold M with boundary by identification of points on the boundary components of M. Then one can define the C^*-algebra

$$\mathcal{A}_{T^*M} = \left\{ \begin{array}{c} u \in C_0(B^*M \setminus S^*M), \\ v \in C_0([0,1), \overline{\Psi}_p(\partial M)) \end{array} \;\middle|\; \begin{array}{c} u|_{S_t} = \mathrm{smbl}(v(t)), \\ u|_{S_0} = v(0) \end{array} \right\},$$

where B^*M and S^*M are respectively bundles of unit balls and spheres in T^*M, $\overline{\Psi}_p(\partial M)$ is the closure of the algebra of parameter-dependent pseudodifferential operators on ∂M, while $S_t \subset B^*M|_{\partial M}$ is the bundle of spheres of radius $t \in [0,1]$.

Denote by $\overline{M}$ the manifold with conical points corresponding to M.

Theorem 3. $\mathrm{Ell}(\overline{M}) \simeq K_0(\mathcal{A}_{T^*M})$.

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On some class of hypersurfaces with pseudosymmetric Weyl tensor

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Abstract. We present results on hypersurfaces with pseudosymmetric Weyl tensor immersed isometrically in semi-Riemannian spaces of constant curvature. The main results are related to the case when such hypersurfaces have at every point at most three distinct principal curvatures.

Translation Continuous Functionals on $CB(\mathbb{R})$ and Their Supports

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There are well-known complications in harmonic analysis of bounded continuous functions on a noncompact locally compact group G (even if this group is amenable [1,2]) related to the fact that the operation of translation on the Banach space formed by these functions is discontinuous [3,4]. In this connection, the study of the space of functionals on $CB(G)$ that are translation continuous is of special interest. Some preliminary results in this direction were obtained in [5]. Here we present two constructions of translation continuous functionals and study properties of these functionals related to their supports. For the simplicity of our exposition, here we restrict ourselves to the most important model case of the additive group $\mathbb{R}$. For the detailed presentation, see [9].

4 Main notions

Let $\mathrm{C_B}(\mathbb{R})$ be the Banach space of real continuous bounded functions on $\mathbb{R}$ (with the usual sup-norm) and ordinary lattice operators. Let $\mathrm{U_B}(\mathbb{R})$ be the space of uniformly continuous bounded functions on $\mathbb{R}$ with the sup-norm of the space $\mathrm{C_B}(\mathbb{R})$, let $\mathrm{C_0}(\mathbb{R}) = \{f \in \mathrm{C_B}(\mathbb{R}) \mid \lim_{x\to\infty} f(x) = 0\}$, and let $\mathrm{L_1}(\mathbb{R})$ be the group algebra with respect to the convolution. Let $\mathrm{C_B}(\mathbb{R})'$ be the dual space of $\mathrm{C_B}(\mathbb{R})$; set $F \leqslant G$ ($F, G \in \mathrm{C_B}(\mathbb{R})'$) if $F(f) \leqslant G(f)$ for any nonnegative function $f \in \mathrm{C_B}(\mathbb{R})$. Thus, a functional $F \in \mathrm{C_B}(\mathbb{R})'$ is said to be *nonnegative* if $F \geqslant 0$. A functional F is called a *mean* if $F \geqslant 0$ and $F(1) = 1$.

Let $h_n \in \mathrm{C_B}(\mathbb{R})$, $n \in \mathbb{N}$, be the function that is equal to 1 on $[-n; n]$ and to 0 outside $(-n-1; n+1)$ and let h_n be linear on $[-n-1; -n]$ and $[n; n+1]$. A functional $F \in \mathrm{C_B}(\mathbb{R})'$ is said to be *of compact type* if $\lim_{n\to\infty} F(fh_n) = F(f)$ for any function $f \in \mathrm{C_B}(\mathbb{R})$ and *of infinite type* if $F(f) = 0$ for $f \in \mathrm{C_0}(\mathbb{R})$, and hence the set of functionals of infinite type is the polar of the set $\mathrm{C_0}(\mathbb{R})$ with respect to the duality $\langle \mathrm{C_B}(\mathbb{R}), \mathrm{C_B}(\mathbb{R})' \rangle$.

Theorem 1. *Every functional $F \in \mathrm{C_B}(\mathbb{R})'$ can uniquely be represented in the form $F = F_1 + F_2$, where $F_1, F_2 \in \mathrm{C_B}(\mathbb{R})'$, F_1 is of compact type, and F_2 is of infinite type. Moreover, $F_1(f) = \lim_{n\to\infty} F(h_n f)$ for any $f \in \mathrm{C_B}(\mathbb{R})$.*

For a chosen $x \in \mathbb{R}$, we define the *shift* of a function $f \in \mathrm{C_B}(\mathbb{R})$ by the element x by the formula ${}_xf(t) \stackrel{\mathrm{def}}{=} f(x+t)$, and the shift of a functional $F \in \mathrm{C_B}(\mathbb{R})'$ by the element $x \in \mathbb{R}$ by the formula ${}_xF(f) \stackrel{\mathrm{def}}{=} F({}_xf)$ for any function $f \in \mathrm{C_B}(\mathbb{R})$. For any set $U \subset \mathbb{R}$, write $L_U(f) = \mathrm{co}\{{}_xf,\, x \in U\}$ and $L_U(F) = \mathrm{co}\{{}_xF,\, x \in U\}$, where $f \in \mathrm{C_B}(\mathbb{R})$, $F \in \mathrm{C_B}(\mathbb{R})'$, and $\mathrm{co}\{\cdot\}$ stands for the convex hull. Let $\mathfrak{L}(\mathbb{R})$ be the set of invariant means. A functional $F \in \mathrm{C_B}(\mathbb{R})'$ is said to be *translation continuous* on $\mathrm{C_B}(\mathbb{R})$ if $\lim_{t\to 0} F({}_tf) = F(f)$ for any $f \in \mathrm{C_B}(\mathbb{R})$ (or, equivalently, $\lim_{t\to 0} {}_tF = F$ in the weak* topology). Denote by $\mathfrak{CF}(\mathbb{R})$ the space of translation continuous functionals; it is norm closed. Recall [6] that every functional $F \in \mathrm{C_B}(\mathbb{R})'$ has a unique decomposition $F = F^+ - F^-$, where $F^+, F^- \geqslant 0$ and $\inf(F^+, F^-) = 0$.

Theorem 2. *If $F \in \mathrm{C_B}(\mathbb{R})'$ is a functional of compact type, then $F \in \mathfrak{CF}(\mathbb{R})$. If $F \in \mathfrak{CF}(\mathbb{R})$, then $F^+, F^- \in \mathfrak{CF}(\mathbb{R})$.*

5 Existence theorems

Let us present assertions that permit one to construct functionals in $\mathfrak{CF}(\mathbb{R})$. A part of the next theorem related to invariant functionals is an immediate consequence of the Markov–Kakutani fixed-point theorem (see [8], Chap. V, §10, Theorem 6).

Theorem 3. *Let $F \in \mathrm{C_B}(\mathbb{R})'$. The weak* closure of the set $L_{\mathbb{R}}(F)$ contains a (translation) invariant functional m such that $\|m\| \leqslant \|F\|$ and, if F is a mean, then $m \in \mathfrak{L}(\mathbb{R})$. Moreover, for any $\delta > 0$, the weak* closure of the set $L_{[-\delta;\delta]}(F)$ contains a functional $\Phi \in \mathfrak{CF}(\mathbb{R})$ such that $\|\Phi\| \leqslant \|F\|$ and $\|\Phi - {}_x\Phi\| \leqslant (5|x|/\delta)\|F\|$ for any x, $|x| \leqslant \delta$, and, if F is of infinite type or is a mean, then so is Φ.*

This theorem enables one to construct an invariant mean for an arbitrary nonzero nonnegative functional. Now let $A \subset \mathbb{R}$ be a closed set and let M_A be the vector space generated by the point functionals corresponding to the elements of A; we refer to the weak* closure of this space as the space of functionals F *supported by* A (and write $\operatorname{supp} F \subset A$). Under the natural isometric isomorphism between the space $\mathrm{C_B}(\mathbb{R})'$ and the space $\mathcal{M}(\beta\mathbb{R})$ of measures on the Stone–Čech compactification $\beta\mathbb{R}$ of $\mathbb{R}$, corresponding to the functionals support by A are those and only those measures on $\beta\mathbb{R}$ whose supports are contained in the closure $\overline{A}_{\beta\mathbb{R}}$ of the set A in $\beta\mathbb{R}$. In particular, Theorem 3 implies the existence theorems (presented below) claiming that, for any "well-shifted" set, there is an invariant mean (and all the more an element of $\mathfrak{CF}(\mathbb{R})$) supported by this set.

Theorem 4. *If a set $A \subset \mathbb{R}$ closed and if $\bigcap_{i=1}^{n}(A + x_i) \neq \emptyset$ for any $x_1, \dots, x_n \in \mathbb{R}$, then there is an invariant mean m supported by A.*

One can readily derive from the condition of the theorem that there is a functional $F \in \mathrm{C_B}(\mathbb{R})'$ such that $\operatorname{supp}{}_x F \subset A$ for any $x \in \mathbb{R}$. Applying Theorem 3 to this functional, we obtain an invariant mean with the desired properties. The following theorem is proved by using similar tools.

Theorem 5. *Let a set $A \subset \mathbb{R}$ closed, let $\delta > 0$, and let the set $\bigcap_{i=1}^{n}(A + x_i)$ be nonempty and unbounded for any $x_1, \dots, x_n \in [-\delta; \delta]$. Then there is a translation invariant mean $\Phi \in \mathrm{C_B}(\mathbb{R})'$ of infinite type such that Φ is supported by A and satisfies the inequality $\|{}_x\Phi - \Phi\| \leqslant 5|x|/\delta$ for any $|x| \leqslant \delta$.*

6 Description of the set of values of the means of infinite type on a given function $f \in \mathrm{C_B}(\mathbb{R})$

Set

$$M_U(f) = \inf_{g \in L_U(f)} \varlimsup_{x\to\infty} g(x), \quad U \subset \mathbb{R}, \qquad m_U(f) = \sup_{g \in L_U(f)} \varliminf_{x\to\infty} g(x), \quad U \subset \mathbb{R}.$$

One can readily see that the following limits exist:

$$m_0(f) = \lim_{n\to\infty} m_{[-1/n;1/n]}(f), \qquad M_0(f) = \lim_{n\to\infty} M_{[-1/n;1/n]}(f).$$

For any neighborhood U of the origin in $\mathbb{R}$ we have $-\|f\| \leqslant \varliminf_{x\to\infty} f(x) \leqslant m_0(f) \leqslant m_U(f) \leqslant m_{\mathbb{R}}(f) \leqslant M_{\mathbb{R}}(f) \leqslant M_U(f) \leqslant M_0(f) \leqslant \varlimsup_{x\to\infty} f(x) \leqslant \|f\|$. Recall that the set of values of the invariant means on a function $f \in \mathrm{C_B}(\mathbb{R})$ is $[m_{\mathbb{R}}(f); M_{\mathbb{R}}(f)]$ (see, e.g., [7], Theorem 1). The set of values of the translation continuous functionals is not described in such an exhaustive way. However, the following result holds.

Theorem 6. *The set of values of the translation continuous means of infinite type on a function $f \in \mathrm{C_B}(\mathbb{R})$ is contained in $[m_0(f); M_0(f)]$ and contains $(m_0(f); M_0(f))$.*

Using Theorem 6, we can describe the set of functions in $\mathrm{C_B}(\mathbb{R})$ on which all translation invariant means of infinite type vanish.

Theorem 7. *Let $f \geqslant 0$ belong to $\mathrm{C_B}(\mathbb{R})$. The following conditions are equivalent:*
1) $F(f) = 0$ for any mean $F \in \mathfrak{CF}(\mathbb{R})$ of infinite type;

2) for any $\delta > 0$ and $\varepsilon > 0$, one can find numbers $\lambda_i \geqslant 0$ such that $\sum_{i=1}^{n} \lambda_i = 1$ and numbers x_i such that $|x_i| < \delta$ and $\| \sum_{i=1}^{n} \lambda_i ({}_{x_i}f) \| \leqslant \varepsilon$, where the symbol $\| \cdot \|$ stands for the norm in the quotient space $\mathrm{C_B}(\mathbb{R})/\mathrm{C_0}(\mathbb{R})$;

3) there is an $A > 0$ such that $M_{[-A;A]}(f) = 0$.

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The Whitehead torus, the Hudson-Habegger invariant and classification of embeddings $S^1 \times S^3 \to \mathbb{R}^7$

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Abstract. We introduce a new invariant for embeddings of higher-dimensional manifolds into Euclidean spaces. Using this invariant and the Haefliger-Wu (the deleted product) invariant we obtain classification results in the dimension range where the Haefliger-Wu invariant was not known to be complete. In particular, we exhibit a new sharp incompleteness example of the Haefliger-Wu invariant.

Our first concrete result states that the group of piecewise-linear embeddings $S^1 \times S^{4k-1} \to R^{6k+1}$ up to piecewise-linear isotopy is isomorphic to $\pi^S_{2k-2} \oplus \pi^S_{2k-1} \oplus Z$. Generally, we classify piecewise-linear embeddings $S^p \times S^{2l-1} \to R^{3l+p}$. We also present smooth analogues of these results. In particular, we prove that the group of smooth embeddings $S^1 \times S^3 \to R^7$ up to smooth isotopy is isomorphic to $Z \oplus Z \oplus Z_2 \oplus Z_{12}$. A classification of embeddings of 4-manifolds into R^m was earlier known only either for $m \geqslant 8$ or for $m = 7$, simply-connected 4-manifolds and the piecewise-linear case (Wu, Haefliger-Hirsch, Hudson, Boechat-Haefliger). A nice feature of our classification results is that representatives of most isotopy classes are explicitly constructed (using, in particular, higher-dimensional Borromean rings and Whitehead link).

Elliptic operators on manifolds with nonisolated singularities

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We consider elliptic operators on manifolds with edges. A manifold with edges is defined as the quotient space of a smooth manifold M, whose boundary is the total space of a smooth fibration $\pi : \partial M \to X$ (with smooth closed base and fibers) by identifying the points in the fibers. The quotient is a singular space consisting of two strata: the top dimensional stratum of smooth points and a lower dimensional stratum X called the *edge*. Near a point on the edge the space is isomorphic to a neighborhood of the point $(0,0)$ in the product $R^n \times K_\Omega$, where K_Ω is the cone with base Ω — the fiber of π. Typical differential operators of order m on manifolds with edges have the form (in a neighborhood of the singular set):

$$D = \frac{1}{r^m} \sum_{|\alpha|+l \leqslant m} a_{\alpha l}(x,r) \left(ir\frac{\partial}{\partial r} \right)^l \left(-ir\frac{\partial}{\partial x} \right)^\alpha, \tag{1}$$

where x are coordinates on the edge, ω — coordinates on Ω, r — radial coordinate on the cone, while $a_{\alpha l}(x,r)$ are smooth families of differential operators on Ω.

The ellipticity condition (e.g. see [1]) for operators of this type consists of the invertibility of the principal symbol $\sigma(D)$ defined on the compressed cotangent bundle T_0^*M on the smooth stratum and the invertibility of the *edge symbol* $\sigma_\Lambda(D)$, which is an operator family on T_0^*X acting in special weighted Sobolev spaces on the infinite cone K_Ω (for the operator D as in (1) the edge symbol is obtained by freezing the coefficients $a_{\alpha l}(x,r)$ at $r=0$ and formally replacing $-i\partial/\partial x \mapsto \xi$). An elliptic operator is Fredholm in the weighted wedge Sobolev spaces.

For elliptic operators on manifolds with edges, we consider the problem of determining the contributions of the strata to the index formula. For several classes of elliptic operators we obtain the contributions of the strata as homotopy invariant functionals of the corresponding symbols. Let us state one of the index formulas.

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Consider some splitting $\partial T^*M \simeq \pi^* T^*X \oplus T^*\Omega \oplus \mathbf{R}$. Let $\alpha : T^*X \to T^*X$ be a linear involution and $\widetilde{\alpha}$ be an extension of this involution to ∂T^*M by adding the identity in the complementary bundle. One can glue two copies of the Atiyah–Bott-Patodi spaces $D^*M = S(T^*M \oplus \mathbf{R})$ along their boundaries twisting by the involution $\widetilde{\alpha}$. The resulting closed manifold is denoted by N_α.

Theorem 1. *If α is orientation reversing (*$\det \alpha = -1$*) and the principal symbol of an elliptic operator D is equivariant under $\widetilde{\alpha}$ then the index has a decomposition:*

$$\mathrm{ind} D = \frac{1}{2}\left(\mathrm{ind}(\mathcal{H} \otimes \gamma^{-1}_{1/2}(T^*2M) \otimes [\sigma(D)]) + \mathrm{ind}[(\alpha^*\sigma_\Lambda(D))^{-1}\,\sigma_\Lambda(D)]\right),$$

as a sum of the index of an elliptic operator on a closed manifold N_α and an index of an operator on the edge X with operator-valued symbol $(\alpha^*\sigma_\Lambda(D))^{-1}\,\sigma_\Lambda(D)$. Here $\mathcal{H}$ is the Hirzebruch (signature) operator on the oriented manifold N_α, γ_t are the Grothendieck operations in K-theory, while $[\sigma(D)] \in \mathrm{Vect}(N_\alpha)$ is the vector bundle defined by $\sigma(D)$. The index of the first operator can be computed by the Atiyah–Singer index formula:

$$\mathrm{ind}(\mathcal{H} \otimes \gamma^{-1}_{1/2}(T^*2M) \otimes [\sigma(D)]) = \langle ch[\sigma(D)]Td(T^*(2M) \otimes \mathbf{C}), [N_\alpha]\rangle,$$

while the index of the second term can be expressed by the Lukeś theorem [2].

Remark 1. 1. For orientation preserving involutions a similar result is valid for anti-equivariant symbols: $\widetilde{\alpha}^*\sigma(D)|_{\partial M} = \sigma(D)^{-1}|_{\partial M}$;

2. The above stated result remains valid for general edge-elliptic problems with boundary and coboundary conditions along the edge.

There is a topological obstruction to the existence of elliptic edge problems for a given elliptic operator D. We give an explicit formula for this obstruction.

Theorem 2. *Let D be a differential operator with elliptic principal symbol. Then this operator has an elliptic edge problem if and only if its principal symbol satisfies the equality:*

$$\pi_*[\sigma(D)|_{\partial T^*M}] = 0 \in K^1(T^*X),$$

where $[\sigma(D)|_{\partial T^*M}] \in K^1(T^*\partial M)$ is the difference element of the restriction of the principal symbol to the boundary of the compressed cotangent bundle, while π_* is the direct image map induced by the projection $\pi : \partial M \to X$.

The results are a joint work with V.E. Nazaikinskii, A. Savin and B.-W. Schulze [3].

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On spherically symmetric space-times

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A 4-dimensional time-oriented Lorentz manifold M is called a *spherically symmetric space-time* if there is an isometric time-orientation preserving group action $\Phi : SO(3) \times M \to M$ such that the highest dimension of its orbits is 2. Several important examples of spherically symetric space-times are known including the Schwarzschild space-time and also a local theory exists ([H-E], pp 369-720). Results concerning a global theory seems to be scarce ([S-W], p 261). Some basic facts pertainig to a general global theory of spherically symmetric space-times are presented in the lecture.

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Generalized Space–Like Ruled Surfaces of the Minkowski Space $\mathbb{R}^n_1$

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We shall assume that all manifolds, maps, vector fields, etc... are differentiable of class $\mathbb{C}^\infty$. Let $\mathbb{R}^n$ be the n-dimensional vector space. The following symmetric, bilinear and non-degenerate metric tensor is called Lorentz metric on $\mathbb{R}^n$;

$$\langle X, Y\rangle = \sum_{i=1}^{n-1} x_i y_i - x_n y_n \quad , \quad X = (x_1, x_2, ..., x_n), \quad Y = (y_1, y_2, ..., y_n) \tag{1}$$

$\mathbb{R}^n$ together with the Lorentz metric is called the n-dimensional Minkowski space, detoned by $\mathbb{R}^n_1$.

Let M be a surface on the n-dimensional Minkowski space $\mathbb{R}^n_1$. If the induced metric on M is Euclidean metric, then M is called the space-like surface. A curve α in $\mathbb{R}^n_1$ is space-like curve, if $\langle \dot{\alpha}, \dot{\alpha}\rangle > 0$, where $\dot{\alpha}$ is velocity vector of α.

Let $\mathbb{R}^n_1$ be a Minkowski space with Levi-Civita connection $\overline{D}$. The function,

$$\overline{R} : \chi(\mathbb{R}^n_1) \times \chi(\mathbb{R}^n_1) \times \chi(\mathbb{R}^n_1) \longrightarrow \chi(\mathbb{R}^n_1) \tag{2}$$

given by

$$\overline{R}(X, Y) Z = \overline{D}_{[X,Y]} Z - \overline{D}_X \overline{D}_Y Z + \overline{D}_Y \overline{D}_X Z \tag{3}$$

is a (3) tensor field on $\chi(\mathbb{R}_1^n)$ that called the curvature tensor field of $\mathbb{R}_1^n$.

Let M be a m-dimensional semi-Riemannian manifold. The function,

$$R : T_M(p) \times T_M(p) \times T_M(p) \times T_M(p) \longrightarrow \mathbb{R} \tag{4}$$

given by

$$R(X_1, X_2, X_3, X_4) = \langle X_1, R(X_3, X_4) X_2 \rangle \tag{5}$$

is an order 4 covariant tensor field on $\chi(M)$ that is called the Riemannian curvature tensor field of M. The Ricci curvature tensor S of M is symmetric and is given relative to a frame field by

$$S(X, Y) = \sum_{i=1}^{m} \varepsilon_i \langle R(e_i, X) Y, e_i \rangle \ , \quad \varepsilon_i = \langle e_i, e_i \rangle = \pm 1 \tag{6}$$

Let $\{e_1(s), e_2(s), ..., e_k(s)\}$ be a system of orthonormal vector fields which are defined for each point of a space-like curve α in the n-dimensional Minkowski space $\mathbb{R}_1^n$. With this system we span a k-dimensional subspace of the tangent space $T_{\mathbb{R}_1^n}(\alpha(s))$ in each point. This subspace that is denoted $E_k(s)$ is

$$E_k(s) = Sp\{e_1(s), e_2(s), ..., e_k(s)\} \tag{7}$$

We get a $(k+1)$-dimensional Lorentz submanifold in $\mathbb{R}_1^n$ if the subspace $E_k(s)$ moves along the curve α. We call this lorentz submanifold an $(k+1)$-dimensional generalized space-like ruled surface in the n-dimensional Minkowski space $\mathbb{R}_1^n$. A parametrization of this ruled surface is as follows

$$\phi(s, u_1, ..., u_k) = \alpha(s) + \sum_{i=1}^{k} u_i e_i(s) \tag{8}$$

Where, $E_k(s) = Sp\{e_1(s), e_2(s), ..., e_k(s)\}$ denotes a space-like subspace, α is a space-like curve and α is an orthogonal trajectory of the k-dimensional generating space $E_k(s)$ $(k \geqslant 1)$. We denote this ruled surface by M.

If we take the partial derivative of ϕ then we get

$$\phi_s = \dot{\alpha}(s) + \sum_{i=1}^{k} u_i \dot{e}_i(s) \tag{9}$$

$$\phi_{u_i} = e_i(s), \qquad 1 \leqslant i \leqslant k. \tag{10}$$

Throughout our paper, we assume that the set

$$\left\{ \dot{\alpha}(s) + \sum_{i=1}^{k} u_i \dot{e}_i(s), e_1, e_2, ..., e_k \right\} \tag{11}$$

is linear independent.

Let M be a $(k+1)$-dimensional space-like ruled surface in $\mathbb{R}_1^n$. The Ricci curvature of M in the direction of vector fields e_r $(1 \leqslant r \leqslant k)$ is given by

$$S(e_r, e_r) = \sum_{j=1}^{n-k-1} \varepsilon_j \left(a_{0r}^j \right), \quad \varepsilon_j = \left\langle \xi_j, \xi_j \right\rangle = \pm 1. \tag{12}$$

The Ricci curvature of M in the direction of vector field e_0 is given by

$$S(e_0, e_0) = \sum_{i=1}^{k} S(e_i, e_i) \tag{13}$$

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Quasi-polyhedrons

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Abstract. The intersection of a finite set of half-spaces of an arbitrary affine space is said to be a quasi-cell in this space. The union K of a finite set of quasi-cells in the affine subspace genetated by K in the affine space is said to be a quasi-polyhedron. Theorem on decomposition of any quasi-polyhedron on a quasi-cell complex will be presented in the paper.