SINGULAR FOLIATIONS WITH EHRESMANN CONNECTIONS

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The concept of Ehresmann connection for a foliation was introduced by Blumenthal and Hebda [1] as a natural generalization of Ehresmann connection for submersions. We have extended this concept on foliations $(M, \mathcal{F})$ with singularities in sense of Stefan and Susmann. Complete singular Riemannian and totally geodesic foliations have natural Ehresmann connections.

We have defined an Ehresmann connection Q of singular foliation $(M, \mathcal{F})$ as a generalized distribution Q on M, which is transverse to $\mathcal{F}$, and a vertical-horizontal property is satisfied. This property allows to transfer an integral curve σ of Q (called horizontal) along admissible curves (called vertical) in the leaf $L = L(\sigma(0))$ of $\mathcal{F}$. Unlike regular case this transfer is not unique in general. For singular Riemannian foliation this transfer keeps lengths of horizontal curves. We used this property and proved a criterion of local stability of leaves of singular Riemannian foliations.

By singular Ehresmann foliation $(M, \mathcal{F}, \mathcal{Q})$ we mean a singular foliation $\mathcal{F}$ with an Ehresmann connection Q. We have introduced a concept of $*Q$-holonomy group of the singular Ehresmann foliation $(M, \mathcal{F}, \mathcal{Q})$ [3].This group has a global character.

On some natural assumptions we have proved that the existence of a compact regular leaf L with a finite holonomy group $*H_Q(L)$ implies compactness of each leaf L_α of the foliation $(M, \mathcal{F}, \mathcal{Q})$ and finiteness of the holonomy group $*H_Q(L_\alpha)$.

Using $*Q$-holonomy groups we have introduced a topological groupoid $*G_Q(F)$ of a singular Ehresmann foliation $(M, \mathcal{F}, \mathcal{Q})$. An important advantage of the groupoid $*G_Q(F)$ is that its topological space is always Hausdorff. In the case of regular foliation this groupoid is equal to the graph $G_Q(F)$ [2].

References

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