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A proof of the Laitinen Conjecture

For a finite group G, consider the subset $Sm(G)$ of the real representation ring $RO(G)$ consisting of the differences $U - V$ of real G-modules U and V which are Smith equivalent, i.e., there exists a smooth action of G on a sphere S such that the fixed point set $S^G = \{x, y\}$ and as G-modules, $T_x(S) \cong U \oplus W$ and $T_y(S) \cong V \oplus W$ for a real G-module W. In 1960, P.A. Smith asked is it true that $Sm(G) = 0$? Now, consider the subset $LSm(G)$ of $Sm(G)$ by imposing the restriction that the action of G on S is such that the H-fixed point set S^H is connected for every cyclic subgroup H of G of order 2^n with $n \geq 3$. In 1996, E. Laitinen posed a conjecture that for a finite Oliver group G, $LSm(G) \neq 0$ if and only if $a_G \geq 2$, where a_G is the number of the real conjugacy classes of G represented by elements of G not of prime power order. By using equivariant surgery and character theory of finite groups, we present a proof of the Laitinen Conjecture and as an application of the result, we answer the original Smith question about $Sm(G)$ to the effect that $Sm(G) \neq 0$ for any finite Oliver group G with $a_G \geq 2$. By using a classification of finite Oliver groups G with $a_G = 0$ or 1, we obtain that if G is simple, $Sm(G) \neq 0$ if and only if $a_G \geq 2$.