

The works of Charles Ehresmann on connections: from Cartan connections to connections on fibre bundles, and some modern applications

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April 5, 2005

In a series of three papers published in *Ann. Ec. Norm.* (vol. 40, 1923, pp. 325–412, vol. 41, 1924, pp. 1–25 and vol. 42, 1925, pp. 17–88), Élie Cartan introduced affine connections on manifolds and defined the main related concepts: torsion, curvature, holonomy groups, . . . He discussed applications of these concepts in Classical and Relativistic Mechanics; in particular he explained how parallel transport with respect to a connection can be related to the principle of inertia in Galilean Mechanics and, more generally, can be used to model the motion of a particle in a gravitational field. In subsequent papers, Élie Cartan extended these concepts for other types of connections on a manifold, called conformal, Euclidean and projective connections.

Around 1950, Charles Ehresmann introduced connections on a fibre bundle and, when the bundle has a Lie group as structure group, connection forms on the associated principal bundle, with values in the Lie algebra of the structure group. He called *Cartan connections* the various types of connections on a manifold previously introduced by É. Cartan, and explained how they can be considered as special cases of connections on a fibre bundle with a Lie group G as structure group: the standard fibre of the bundle is then an homogeneous space G/G' ; its dimension is equal to that of the base manifold; a Cartan connection determines an isomorphism of the vector bundle tangent to the the base manifold onto the vector bundle of vertical vectors tangent to the fibres of the bundle along a global section.

These works will be reviewed, and some applications of the theory of connections in modern physics will be sketched.