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Perfectness at infinity of diffeomorphism groups on open manifolds

Let us recall that a group G is called perfect if $G = [G, G]$, where the commutator subgroup is generated by all commutators $[g_1, g_2] = g_1 g_2 g_1^{-1} g_2^{-1}$, $g_1, g_2 \in G$. In terms of homology of groups this means that $H_1(G) = G/[G, G] = 0$.

It is well known that the identity component of the group of all compactly supported C^r-diffeomorphisms of a manifold is perfect and simple provided $1 \leq r \leq \infty$, $r \neq n + 1$, and n is the dimension of the manifold (theorems of Herman, Thurston and Mather). Several generalizations for the automorphism groups of geometric structures are known. The problem of the perfectness of analogous groups with no restriction on support is studied by making use of results of Segal. It is only very loosely related to the problem of the perfectness of compactly supported diffeomorphism groups. A clue role in the problem plays the notion of the perfectness at infinity. We show that the identity component of the group of all C^r-diffeomorphisms on $\mathbb{R}^n$ is perfect at infinity. Also we formulate some conditions which ensure together with the perfectness at infinity that the diffeomorphism group in question is perfect.