Existence of natural and projectively equivariant quantizations

Sarah Hansoul
University of Lični¿½ge
12, Grande Traverse,B-4000 Ličge
email : s.hansoul@ulg.ac.be

In this talk, what we call a quantization is a linear bijection

$$Q_\nabla : \mathcal{S}(M) \to \mathcal{D}(M),$$

where $\mathcal{D}(M)$ is a space of differential operators and $\mathcal{S}(M)$ its associated space of symbols. This map depends on a linear connection ∇ on a differential manifold M.

A few years ago, P. Lecomte conjectured the existence of a *natural and projectively equivariant quantization*, namely a quantization which is natural and takes the same values on two connections projectively equivalent. The existence of such a quantization is a generalization on an arbitrary manifold of the existence of the so-called sl_{m+1}-*invariant quantization* previously obtained on R^m.

We obtain a sufficient condition for the existence of such quantizations, when the differential operators considered act between sections of vector bundles associated to the fiber bundle P^1M of linear frames of M. In this proof, we use both approaches of projective structures due to T.Y. Thomas and J.H.C. Whitehead in the 1920's. In particular, we use the theory of Cartan connexions, and construct a Casimir operator depending on a Cartan connexion. We show the existence of a quantization when the eigenvalues of this operator are of multiplicity one.