Groupoids, Local-to-global, Higher dimensions:
Three themes in the work of Charles Ehresmann

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ABSTRACT: Charles Ehresmann's work on category theory is unusual in its emphasis on the relations between local and global methods and results, often with the notion of differentiable (or Lie) groupoids, and for early ventures into higher order categories, through notions of structured categories.

This talk will show two of his strong influences on my work.

One, obtained via Jean Pradines, is the use of admissible local sections of a locally Lie groupoid. The background work of Ehresmann on Lie groupoids and germs allowed for an interesting algebraic encapsulation of the intuition of 'iteration of local procedures', and so to obtain a holonomy groupoid, and then a monodromy groupoid, in situations more general than foliations.

Second, Ehresmann's initial work on double categories was published when I was trying to express a putative higher order van Kampen theorem in terms of conjectured higher homotopy groupoids. These were found with Philip Higgins in the late 1970s. They yielded nonabelian methods for certain local-to-global problems in dimensions more than 1, and a new version of the interface between homology and homotopy.

Some work has been done on the interaction of these two themes, in which local admissible sections are generalised to certain local homotopies of an identity morphism to an automorphism.

Much more work is needed to apply these methods in the theory and applications of manifolds.