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On SO(3) geometry in dimension five (joint work with P. Nurowski)

A nonstandard inclusion $\mathbf{SO}(3) \subset \mathbf{SO}(5)$ associated with the irreducible representation ρ of $\mathbf{SO}(3)$ in $\mathbf{R}^5$ is considered. The tensor t reducing the group $\mathbf{O}(5)$ to this $\mathbf{SO}(3)$ is found. It is given by a ternary symmetric form with some special properties. A 5-dimensional manifold (M, g, t) with the Riemannian metric g and the ternary form t generated by such tensor defines an $\mathbf{SO}(3)$ structure on M. The Gray-Hervella-type classification of such structures is established using the $\mathbf{so}(3)$-valued connections with torsion. Structures with antisymmetric torsions are studied in detail. In particular, it is shown that the integrable models (those with vanishing torsion) are isometric to the symmetric spaces $M_+ = \mathbf{SU}(3)/\mathbf{SO}(3)$, $M_- = \mathbf{SL}(3,\mathbb{R})/\mathbf{SO}(3)$, $M_0 = (\ \mathbf{SO}(3) \times_\rho \mathbb{R}^5)/\mathbf{SO}(3)$. We also find all $\mathbf{SO}(3)$ structures with transitive symmetry groups. Given an $\mathbf{SO}(3)$ structure (M, g, t) we define its twistor space Z to be an $\mathbf{S}^2$-bundle of those 2-forms on M which span a 3-dimensional irreducible representation of $\mathbf{SO}(3)$ and which have unit length. The 7-dimensional twistor manifold Z is then naturally equipped with several CR-structures and G_2-structures. The integrability conditions for these structures are discussed and interpreted in terms of the Gray-Hervella-type classification of (M, g, t).